Informational Substitutes

Yiling Chen
Harvard

Bo Waggoner
UPenn
0. Information
Random variables $X, Y_1, \ldots, Y_n$ jointly distributed, known prior. (finite set of outcomes)

We care about $X$.

$Y_i = \text{“signal”} \ (\text{reveals info. about } X)$. 

![Diagram showing the relationship between random variables $X$, $Y_1$, $Y_2$, and $Y_3$ and their dependence on nature event $X$.](image)
1. Substitutes
The unreasonable effectiveness of substitutes

Substitutes in economics:
- Market equilibria, stable matchings, ...

Substitutes in computer science:
- Submodularity! [Lehmann+Lehmann+Nisan 2001]
- subs == efficient approx. for many problems

Could we also define “substitutes” for information?

And could they also link algorithms and game theory?
## Challenges for defining informational S&C

<table>
<thead>
<tr>
<th>Items</th>
<th>Information</th>
</tr>
</thead>
<tbody>
<tr>
<td>● Valuation function $f$ is given</td>
<td>● What is the “value” of a set of pieces of information?</td>
</tr>
<tr>
<td>● $f(S)$ does not depend on $f(T)$</td>
<td>● Two pieces of information may be correlated, redundant, ...</td>
</tr>
</tbody>
</table>

...
2. Story
What is value of information?
The Story part 1: Shannon 1948

- prior $p$ on $X$
- posterior given $Y=y$

$H(X) - H(X \mid Y)$
1. Known prior $p$ on $X$

2. Select decision $d$

2. Nature draws $x \sim p$

3. Get utility $u(d, x)$.

$V(\emptyset) = \text{“expected utility when deciding optimally with no signals”}$
The Story part 2: Howard 1966

1. Known prior \( p \) on \( X \)

1.5. **Observe \( Y \), Bayesian update to \( p_y \)**

2. Select decision \( d \)

2. Nature draws \( x \sim p_y \)

3. Get utility \( u(d, x) \).

\[ V(Y) = \text{“expected utility when deciding optimally after observing } Y \text{”} \]
The Story part 2: Howard 1966

1. Known prior \( p \) on \( X \)

1.5. **Observe \( Y \)**, Bayesian update to \( p_y \)

2. Select decision \( d \)

2. Nature draws \( x \sim p_y \)

3. Get utility \( u(d, x) \).

\[ V(Y) = \text{“expected utility when deciding optimally after observing } Y\text{”} \]

\[ V(Y) - V(\emptyset) = \text{“marginal value of } Y\text{”} \]
1. Known prior \( p \) on \( X \)

2. Select prediction \( q \)

2. Nature draws \( x \sim p \)

3. Get utility \( S(q, x) \).

“Proper scoring rule” - optimal prediction is true belief
Example: $S(q, x) = \log q(x)$. 
Savage: scoring rules $\leftrightarrow$ convex functions

Example: $S(q, x) = \log q(x)$. 

Expected score as function of belief for binary $X$
Savage: scoring rules $\leftrightarrow$ convex functions

Example: $S(q, x) = \log q(x)$.

- prior $p$ on $X$
- posteriors $p_y$

$V(Y) - V(\emptyset)$

Expected score as function of belief for binary $X$
DECISION PROBLEMS $\leftrightarrow$ convex functions!

Example: $S(q, x) = \log q(x)$.

- prior $p$ on $X$
- posteriors $p_y$

$V(Y) - V(\emptyset)$

Expected score as function of belief for binary $X$
3. Definitions
Visualizing substitutes for log scoring rule

Example: $S(q, x) = \log q(x)$.

- prior $p$ on $X$
- posteriors $p_y$
- $V(Y) - V(\emptyset)$
Visualizing substitutes for log scoring rule

Example: $S(q, x) = \log q(x)$.

- prior $p$ on $X$
- posteriors $p_y$
- $V(Y) - V(\emptyset)$
- posteriors $p_{yz}$
- $V(Y,Z) - V(Y)$

Expected score for predicting $X$ as function of posterior belief
Our definitions

$Y_1...Y_n$ are substitutes for $u$ if $V$ is submodular:
For $A \subseteq B \subseteq \{Y_1...Y_n\}$,

$$V(A \cup \{Y_i\}) - V(A) \geq V(B \cup \{Y_i\}) - V(B).$$

- complements = supermodular
- depends on both decision prob AND info structure
Roadblock: Information is divisible!

“Half the truth is often a great lie.”
- Benjamin Franklin
Roadblock: Information is divisible!

“Half the truth is often a great lie.”
- Benjamin Franklin

Solution: extend definitions.
(See my TCS+ talk on Nov. 9!)
Two separate applications:

- Markets (for information).
  substitutes \(\leftrightarrow\) good equilibria

- Algorithms.
  complexity of optimal info. acquisition
4. Prediction markets
Idea / motivation

Each agent has a signal $Y_i$.

Goal: aggregate into prediction about $X$ quickly.
Idea / motivation

Each agent has a signal $Y_i$.

Goal: aggregate into prediction about $X$ **quickly**.
The mechanism [Hanson 2003]*

Only one participant: proper scoring rule! Truthful.

*can also be viewed as buying/selling shares [Abernethy+Chen+Wortmann-Vaughan 2013]
The mechanism [Hanson 2003]

Two participants: “chained” scoring rule! Truthful.
The mechanism [Hanson 2003]

Two participants, three stages: **not understood!**
Two participants, three stages: **not understood**!

Known: for **log scoring rule**, if $Y_1 \ldots Y_n$ are

- conditionally independent on $X \Rightarrow$ “rush”.
  [Chen+Dimitrov+Sami+Reeves+Pennock+Hanson+Fortnow+Gonen 2010]

- independent $\Rightarrow$ “delay”.
  [Gao+Zhang+Chen 2013]
Prediction markets results

**Thm.** If and only if signals are strong **substitutes**, the only equilibria are “all rush”.

(efficient market hypothesis $\leftrightarrow \text{substitutes}$)

**Thm.** If and only if signals are strong **complements**, the only equilibria are “all delay”.

(market failure $\leftrightarrow \text{complements}$)
5. Algorithms
Algorithmic question “SignalSelection”

Input:
- utility function $u$ (as an oracle...)
- joint distribution $X, Y_1 \ldots Y_n$ (as an oracle...)
- prices $\pi_1 \ldots \pi_n$ for the signals, budget constraint $B$

Output:
- which signals to acquire
Reduction: SignalSelection $\rightarrow$ set function maximization.
Substitutes $\Rightarrow$ $1-1/e$ approx in polynomial time

Reduction: set function maximization $\rightarrow$ SignalSelection.
Comps/generally $\Rightarrow$ no approx w/ subexp. queries

Notes:
- As in submod. maximization, can handle e.g. matroid constraints.
- Ideas not new here at all! See [Krause+Guestrin 2011]
- Model / generality, focus of our question are new
6. Shadows
7. Possibilities
Open problems (small selection)

Game theory
- selling information
- signalling
- bundling complements
- other useful applications?

Algorithms
- check if signals are strong substitutes
- compute Alice’s best response in stage one
  (decompose signal into sub. and comp. components)
- SignalSelection on discrete or continuous lattices!

Structure
- examples of (classes of) subs and comps
- “more substitutable” signal structures? utilities?
- “universal” substitutes and complements
- connections: e.g. sensitivity of Boolean functions?
Resources

TCS+, my talk on **Nov. 9**

sites.google.com/site/plustcs/

These slides:

bowaggoner.com/

Blog posts on proper scoring rules, generalized entropies, ...

bowaggoner.com/blog/

Information elicitation slides:

sites.google.com/site/informationelicitation/

Thanks!
extra slides
CALL FOR PAPERS

57th Annual IEEE Symposium on Foundations of Computer Science (FOCS 2016)

New Brunswick, New Jersey, October 9-11, 2016.

The 57th Annual Symposium on Foundations of Computer Science (FOCS 2016), sponsored by the IEEE Computer Society Technical Committee on Mathematical Foundations of Computing, will be held in New Brunswick, New Jersey on October 9-11 (Sunday through Tuesday).

On Saturday, October 8th, FOCS will join the celebration of Avi Wigderson's 60th birthday.

Papers presenting new and original research on theory of computation are sought. Typical but not exclusive topics of interest include: algorithms and data structures, computational complexity, cryptography, computational learning theory, economics and computation, parallel and distributed algorithms, quantum computing, computational geometry, computational applications of logic, algorithmic graph theory and combinatorics, optimization, randomness in computing, approximation algorithms, algorithmic coding theory, algebraic computation, and theoretical aspects of areas such as networks, privacy, information retrieval, computational biology, and databases. Papers that broaden the reach of the theory of computing, or raise important problems that can benefit from theoretical investigation and analysis, are encouraged.