On Proper Losses for Evaluating Discrete Generative Models



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DIMACS October 19, 2023

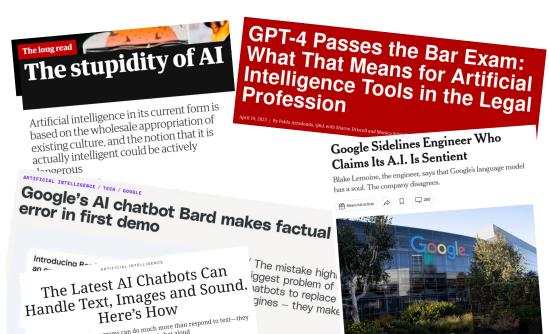
This talk:

- **1** Motivation: importance of evaluation
- **2 Research:** proper losses for generative models
- **Future:** types of tasks

1. Motivation

Q: How good are LLMs?

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All image credits: Midjourney







Octopoids to Make Schoolteachers Obsolete



As engineering?

As engineering?

OpenAI Bridge Supports Elephant Herd



As engineering?



An evaluation crisis

Problems:

- ML research incentives: new and shiny achievements
- Industry incentives: . . .

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Benefits of evaluation research

Rigorous understanding of strengths and weaknesses

not hope

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- ... leading to fundamental progress
- Improved training methods
- Honest public relations

No snake oil; no winter

not hope

2. Research

Proper losses

Proper Losses for Discrete Generative Models, ICML 2023.



Dhamma Kimpara CU Boulder



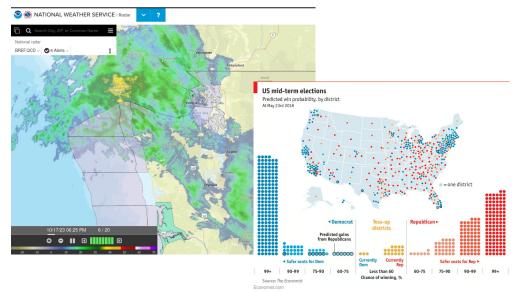
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Motivation: forecasting

Example: forecast a weather system trajectory, or an election



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Goal: generative model should match reality as closely as possible. *Similar: GANs*

Background

Traditional proper loss: ℓ (prediction, outcome) such that $\mathbb{E}_{y \sim q} \ell(p, y)$ is minimized by predicting p = q. a.k.a. proper scoring rule

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Key examples:

Squared loss,
$$\ell(p, y) = \|p - \delta_y\|_2^2$$
a.k.a Brier score
 Log loss, $\ell(p, y) = \log(1/p_y)$
a.k.a cross entropy a.k.a cross entropy

Lots of research in supervised learning: consistency, calibration, etc

Generative models

Problem: generative models are (often) black boxes.

 \implies cannot generally query p_y .

or not easy, efficient

 \implies cannot calculate loss $\ell(p, y)$.

Recall: $||p - \delta_y||_2^2$, $\log(1/p_y)$.

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- \implies cannot calculate loss $\ell(p, y)$. Recall: $\|p \delta_y\|_2^2$, $\log(1/p_y)$.

Their only interface (suppose): press button, generate example

Proposal

Let p be a model and q a ground truth distribution.

```
We draw samples A \sim p and B \sim q.
```

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The loss is **black-box proper** if, for all q, $\mathbb{E}[\ell(A, B)]$ is minimized by choosing p = q.

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(n,m) black box loss:

- A is n iid draws from p (the model)
- B is m iid draws from q (the world).

Main result

Theorem

For any $n \ge 2$ and any $m \ge 1$, there exists an (n,m) black-box strictly proper loss.

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Furthermore, ℓ is strictly black-box proper $\iff g(p,q) := \mathbb{E}[\ell(A,B)]$ is a polynomial in p and q of degree at most n and m resp. such that, for all q, the minimizer of g is p = q.

Furthermore, we can construct ℓ from g using theory of unbiased estimators.

Key example: squared loss.

Naive attempt:
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empirical distributions

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Bonus: By drawing Poisson, can also implement **log loss** via Taylor series.

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Problem: in high-dimensional spaces, "signal" is rare *lower bounds for distribution learning*

When these losses are practical: on low-dimensional features

- **Language:** sentence lengths, other statistics
- Images: autoencoder-type features
- Structured output: low-dimensional summaries

Could search for a feature with high loss, a la GANs

3. Future



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but dimensionality challenges



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- When can we frame these as forecasting?
- Contrast: game-playing
- Contrast: zero-knowledge proofs

cf Yogi Berra

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Thanks!

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