## Where Information and Incentives Collide



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## Outline

1 Contracts
2 Public projects
3 Matching
Themes

- Gathering hidden information
- Navigating preferences and strategic behavior
- Coordinating good decisionmaking
- Solving algorithmic problems in societal contexts


## Contracts



Joint work with Maneesha Papireddygari
The 2022 ACM Conference on Economics and Computation (EC '22)

## Eliciting predictions

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- An expert makes a prediction $p$
- We observe whether the event happened, $y$
- We assign a score or payment $S(p, y)$



## Characterization of proper scoring rules

## Fact (McCarthy 1956; Savage 1971; ...)

A scoring rule is proper (meaning truthful) if and only if it is

$$
S(p, y)=G(p)+\nabla G(p) \cdot\left(\delta_{y}-p\right)
$$

for some convex $G$.


## Next problem: information acquisition

What if the expert can acquire costly information?


How do we incentivize truthful, accurate predictions?

## Next problem: hidden action (moral hazard)

How to write contracts to incentivize good, unverifiable work?

- Long history in economics
- Recent algorithmic work in CS/Econ


## Contract $t$



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## What about both at once?

Final problem: hidden action with information acquisition.


## Overview of problems



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## Key insight



## Results

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S ("no leak", "no leak") $\longrightarrow$
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For the information acquisition problem, there is a closed-form solution (an inverted pyramid).

## Connections and takeaways

- actions $\leftrightarrow$ predictions
- value of information
- framing contract design as information elicitation



## Public Projects



Joint work with Mary Monroe,
in preparation
Funding: The Ethereum Foundation (2022-)

## Public Projects

A classic problem: a bunch of people want to decide what to do together.


## A mathematical model. . .

Each person has a value for each alternative


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Goal: maximize social welfare of project


## Existing solutions

Classical solution: a "VCG mechanism".
Maximizes welfare, but...

- Fragile: false-name attacks, ...
- Unpredictable: payments may be zero, very high, in between...
- No revenue: often, nobody pays anything


## Using quadratic voting

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- Each person casts "votes" for (or against) each option
- Pay $c$ times the number of votes, squared
- Pick the winner with "softmax":

$$
\operatorname{Pr}[\text { select project } j]=\frac{e^{\text {total votes for } j}}{\sum_{k} e^{\text {total votes for } k}}
$$

## Prior work

Theorem (Eguia et al. 2022): if participants' preferences are drawn i.i.d. with bounded values, then in any symmetric Bayes-Nash equilibrium,

$$
\operatorname{Pr}[\text { select outcome with maximum social welfare }] \rightarrow 1
$$

as
num. participants $\rightarrow \infty$.

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## Proposition

A pure-strategy equilibrium exists if $c>\frac{3}{2} \max _{i, k}\left|u_{k}^{i}\right|$.

## Proof ingredients

- analyze Hessian of utility function
related: exponential families
- fixed-point theorem for concave utilities
- properties of $x e^{-x}$



## Results continued

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With $m$ choices, in any pure-strategy equilibrium,

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\frac{\text { Social Welfare }}{\text { Optimal SW }} \geq 1-f\left(c, U_{1}, \ldots, U_{m}\right)
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where we can write down $f$, but it ain't pretty.

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## Conjecture

1 If all participants agree on the ordering of the alternatives, a pure-strategy equilibrium always exists.
2 In mixed-strategy equil., SW remains high under many conditions.

## Future work

- Explore limits of this mechanism
- Explore connections to prediction and decision markets



## Matching



## Joint work with Robin Bowers,

to appear in the 2023 Conference on Web and Internet Economics (WINE)

Funding: The National Science Foundation (2023-)

## Matching

Classic problem: how to match e.g. workers to jobs?


## Max-Weight Matching

One formulation: maximize total value of the matching.


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Model: each person has a distribution over possible values for each job, and a cost for finding out.


## Simplified version of the problem

- Matching people to items



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- Matching people to items
- Selling one item



## The Pandora's Box Problem

Due to Weitzman (1979)
also a case of Gittins index thm.

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Optimal "descending policy":

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Application: selling one item [Kleinberg, Waggoner, Weyl (EC 2016).]

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Application: selling one item [Kleinberg, Waggoner, Weyl (EC 2016).]
Idea: mimick the optimal policy with a descending-price auction.
Result: constant-factor approximation to optimal social welfare.
Observation: failure of ascending-price; and of any sealed-bid auction.

# Back to our problem: two-sided matching 

Proposal: the Marshallian Match. ${ }^{1}$
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- Participants maintain bids on all potential partners.

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- Participants maintain bids on all potential partners.
- A global price descends over time.
- When the price reaches the total bid on an edge, it matches.
- Both sides pay their bids.

[^3]
## Results (1)

## Theorem

If all participants' values are positive, the Marshallian Match guarantees, in any Bayes-Nash equilibrium,

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\frac{\text { Social Welfare }}{\text { Optimal SW }} \geq \frac{1}{8}
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- Holds for model with inspection costs
- Also holds for matchings on hypergraphs (group formation) factor depends on maximum group size


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Ingredient 1: greedy max-weight matching
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Ingredient 4: Rebate variant of Match: align incentives with early matching

Ingredient 5: Information-hiding allows counterfactual analysis

## Results (2)

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In general settings with common-knowledge values, if player strategies are 2-ex-ante stable, the Marshallian Match guarantees

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- Ex-ante stability: extension of equilibrium to pairs of players
- Proven for model without inspection costs; may extend
- Unknown: extends to Bayes-Nash setting? (main open problem)

Outro

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Other exciting work going on in algorithmic economics and theoretical ML group²: Prof. Raf Frongillo; Ph.D. students Dhamma, Anish, Maneesha, Rick, Robin, Mary, Melody, Elias; theory group: Prof. Josh Grochow, Prof. Huck Bennett, students, ....


[^4]


Thanks!


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[^4]:    ${ }^{2}$ [JF, RF, BW (COLT 2020, NeurIPS 2021, JMLR 2023)] [RF, BW (NeurIPS 2021)] [RF, AG, AT, BW (EC 2021)] [DK, RF, BW (ICML 2023)].

