UPenn NETS 412: Algorithmic Game Theory
Homework 1

Instructor: Bo Waggoner
Due: by beginning of class, January 25, 2018
Turn in *electronically* via Gradescope.

Problem 1 (5 points)

Consider a one-shot, two-player game in which Player 1 (the row player) has two strategies, T and B, and Player 2 (the column player) has three strategies, L, C, and R. Player 2 observes that, regardless of whether Player 1 plays T or B, playing R is never a best-response.

True or false: at least one of L and C must at least weakly dominate R for Player 2. If you say True, explain why. If you say False, provide a counterexample.

		Player 2		
		L	C	R
Player 1	T	?	?	?
	B	?	?	?

Problem 2 (8 points)

A Stag Hunt game is a one-shot, two-player, two-action game in which the players can decide to either hunt a stag (S) or a hare (H). Consider the Stag Hunt game below for the following problem. Throughout, let p denote the probability that Player 1 plays S and q the probability that Player 2 plays S.

		Player 2	
		S	H
Player 1	S	$6, 6$	$0, 2$
	H	$2, 0$	$3, 3$

Part a (2 points) Assuming that Player 2 is playing S with probability q^*, what is Player 1's expected payoff for playing S? Write your answer as a function of q^*.

Part b (2 points) Assuming that Player 2 is playing S with probability q^*, what is Player 1's expected payoff for playing H? Write your answer as a function of q^*.

Part c (2 points) What is Player 1's best-response function? That is, how should she pick p as a function of Player 2's choice of q? Write your answer as a function of q.

Part d (2 points) What are the Nash equilibria of this game?

Problem 3 (8 points)

Consider an n-player game ($n \geq 2$) where each player's must choose a number from $[0, 100]$ as her 'guess' for what the other players will choose. The closest guess to the $2/3 \cdot \mu$ (where μ is the average of all n guesses) "wins" and obtains utility 1; all others get utility 0. (Ties are broken arbitrarily.)

Part a (2 points) Find a Nash equilibrium and prove that it is an equilibrium.

Part b (3 points) Does there exist a Nash equilibrium in dominant strategies?

Part c (2 points) Do your answers change if guesses can only be integral (i.e. from $\{0, 1, \ldots, 100\}$)? Why or why not?

Part d (1 point) If you were playing this game in real life, would you play according to your predictions? Why or why not?

Problem 4 (5 points)

In Lecture 1 Claim 5 we state that if iterated elimination of dominated strategies results in a unique solution, then it is a pure strategy Nash equilibrium. Prove that if only a single strategy profile s survives iterated elimination of *weakly* dominated strategies (*i.e.* if at the end for all i, $|B_i^t| = 1$ and $s_i \in B_i^t$ is the surviving action of player i) then s is a pure strategy Nash equilibrium of the game.

Problem 5 (10 points)

Consider a congestion game with two facilities and n players. Both facilities have the same cost function given by the identity function, i.e. the cost of using a facility is the total number of players that are using that facility. Each player must choose exactly one facility to use.

Part a (4 points) Define a potential function for this game, and show that a single step of best response dynamics cannot increase your potential function.

Part b (2 points) Give an example of an action profile where best response dynamics do not halt, and calculate the change in potential from a single step of the algorithm.

Part c (2 points) At what state does the best response dynamic halt? Calculate the potential at this state.

Part d (2 points) Give a pure-strategy Nash equilibrium for this game.