UPenn NETS 412: Algorithmic Game Theory
Homework 4

Instructor: Bo Waggoner
Due: by beginning of class, March 22, 2018
Turn in *electronically* via Gradescope.

Problem 1 (5 points)

Does the Top Trading Cycles algorithm always produce the same allocation, regardless of which tiebreaking rule we use to pick a cycle to clear? If so, prove it; otherwise, give a simple counterexample.

Problem 2 (5 points)

The Deferred Acceptance algorithm is truthful for the side that proposes, but not necessarily for the side that does not. Give a simple example of a set of preferences such that some participant can improve by misreporting, and explain how this helps that participant change the algorithms' decisions.

Problem 3 (5 points)

(Voting.) Give a polynomial-time algorithm that takes in a list of rankings and decides whether or not there exists a Condorcet winner. If so, your algorithm should output this winner. Argue that it is correct and runs in polynomial time. (You only have to find a candidate that strictly wins all pairwise comparisons.)

Problem 4 (5 points)

(Voting.) The *Borda* voting rule gives each candidate m points for being ranked first by a voter, $m-1$ for being ranked second, $\ldots$, 1 point for being ranked last. It selects as a winner the candidate with the most total points.

Show that the Borda rule computes the maximum likelihood estimate of the winner under the following noise model: *There is a "true best candidate" j^*. Each voter ranks j^* in position k with probability proportional to 2^{-k}, and ranks all other candidates completely at random.*