UPenn NETS 412: Final Practice Problems

April 2018

Problem 1

For this problem, you should know the VCG mechanism as well as Myerson's Lemma characterizing truthful choice functions.

A telecom company wants to decide which neighborhood to give fiber internet access. There are m neighborhoods $1, \ldots, m$. Each user i is located in exactly one neighborhood and has value v_i if her neighborhood gets fiber and value 0 otherwise, where $v_i \in \mathbb{R}_{\geq 0}$. Let N_j be the set of users located in neighborhood j. The telecom company wants to design a truthful mechanism (X, P) for this problem. Here truthful means dominant strategy incentive compatible (DSIC).

For each of the following choice functions X, does there exist a payment rule P such that the mechanism is DSIC? Why or why not? If there does, for a bonus point each, give an example of such a payment rule.

Part a Suppose the telecom company wants to select the neighborhood with the largest sum of values. That is, $X(v) = \arg\max_j \sum_{i \in N_j} v_i$.

Part b Each neighborhood j costs the telecom company a different amount C_j to build infrastructure. So the company wants to select the neighborhood with the largest sum of values minus cost of investment. That is, $X(v) = \arg\max_j \left(\sum_{i \in N_j} v_i\right) - C_j$.

Part c The telecom company wants to give fiber to the neighborhood that is most heterogeneous in valuation, for some reason. So it picks the neighborhood where the difference between highest and lowest value is smallest, i.e. $X(v) = \arg\max_j \left(\max_{i \in N_j} v_i\right) - \left(\min_{i \in N_j} v_i\right)$.

Part d The telecom company wants to give fiber to the neighborhood that is most homogeneous in valuation, for some reason. So it picks the neighborhood where the difference between highest and lowest value is smallest, i.e. $X(v) = \arg\min_j \left(\max_{i \in N_j} v_i\right) - \left(\min_{i \in N_j} v_i\right)$.

Part e The telecom wants to give fiber to neighborhoods that have higher value for it, but it wants to diminish the impact of the high outliers. So it wants to maximize the sum of the logarithms of values, i.e. $X(v) = \arg\max_j \sum_{i \in j} \log(v_i)$.

Problem 2

For this problem, you should know the proper scoring rule characterization as well as peer prediction.

Priscilla hires you to help her elicit predictions. She wants experts to make predictions $p \in [0,1]$ representing the probability of rain the next day. When she then observes $y \in \{\text{rain}, \text{no rain}\}$, she will score them with some function $S(p, y)$.

She asks you to design the scoring rule S. It must be truthful: for any belief q, the optimal report should be q. Next, she will add some additional requirements.

Part a Suppose in this part that Priscilla requires:

- The expected score for an agent who believes $q = 0$ should be 10.
- The expected score for an agent who believes $q = 1$ should be 10.
- The expected score for an agent who believes $q = 0.5$ should be 20.

Tell Priscilla how to design a scoring rule satisfying her requirements, or explain why it is impossible.

Part b Suppose in this part Priscilla requires that the expected score be larger for agents who believe higher probabilities of rain. That is, if $q > q'$, then an agent who believes q should expect to make more money than she would if she believed q'.

Tell Priscilla how to design a scoring rule satisfying her requirements, or explain why it is impossible.

Part c Now Priscilla changes the rules a bit. She knows the expert has observed a piece of information about the weather, a signal which was either "high" or "low". If the expert observed high, then he believes a probability of 0.75 of rain; if low, then 0.25. Priscilla wants you to design a payment rule that just asks the expert to report either high or low, then scores this report with the observed outcome (rain or no rain). It should be a function taking two arguments, the signal and the observation of whether it actually rained or not. It should be truthful, i.e. the agent should strictly prefer reporting his true signal to lying.

Tell Priscilla how to design a payment function satisfying her requirements, or explain why it is impossible.

Part d Suppose we know the expert's initial belief is $q = 0.5$. However, the expert has a crystal ball available that will enable him to find out for sure whether or not it will rain. The catch is that it costs him 5 units of utility to use the crystal ball. Priscilla wants you to design a scoring rule so that the expert's optimal action is use the crystal ball, then report truthfully what he finds out.

Tell Priscilla how to design a scoring rule satisfying her requirements, or explain why it is impossible.

Problem 3

Consider a game in which each of two players announces an integer between 0 and $2n$, where n is fixed ahead of time and known to both players. Let a_1 denote the announcement of Player 1 and a_2 denote the announcement of Player 2. The payoffs are determined as follows:

1. If $a_1 + a_2 \leq 2n$, Player 1 receives a_1 and Player 2 receives a_2.
2. If $a_1 + a_2 > 2n$ and $a_1 > a_2$, Player 1 receives $2n - a_2$ and Player 2 receives a_2.
3. If $a_1 + a_2 > 2n$ and $a_1 < a_2$, Player 1 receives a_1 and Player 2 receives $2n - a_1$.
4. If $a_1 + a_2 > 2n$ and $a_1 = a_2$, each player receives n.

Find an equilibrium of this game using iterated elimination of dominated strategies first when $n = 5$ and then when $n = 50$.

Problem 4

Part a Define *regret* in an expert-advice setting.

Part b Define *average regret* in an expert-advice setting.

Part c Outline the halving algorithm, and report its guarantees in the case of a perfect expert and an expert making OPT mistakes.

Part d Outline the polynomial weights algorithm, and give the best $\mathcal{O}()$ bound you know for its regret and average regret.

Problem 5

Consider the polynomial weights algorithm. In which of the following situations would you most want a high value for ϵ. justify your response:

1. The performance of the actions is relatively uniform
2. The best action is perfect (loss zero) throughout all rounds
3. The best action starts start off poorly, then finishes well

4. The worst action starts off well, then finishes poorly

Problem 6

Consider the Top Trading Cycles (TTC) algorithm. Suppose that an individual's favorite item is the one she starts with. Explain why she is indifferent between participating in the game and not, and explain why her participation does not affect the allocation to all of the other players, regardless of their preferences.

Problem 7

Give an example with three people on each side of the Stable Marriage Problem in which each person has a total, strict preference order over the people on the opposite side and there exists a *unique* stable matching. Show that your matching is indeed unique and demonstrate how the Deferred Acceptance (Gale-Shapley) algorithm finds it by showing the steps the algorithm takes.

Problem 8

Consider the following set of voter preferences over three candidates, A, B, and C, where we write $X \succ Y$ to indicate that X is preferred to Y. There are 13 voters.

1. Four people rank $B \succ A \succ C$
2. Four people rank $C \succ A \succ B$
3. Three people rank $A \succ B \succ C$
4. Two people rank $C \succ B \succ A$

Part a If we use the *plurality rule*, which candidate wins and what is the count of the votes?

Part b Suppose we use the following rule: each person votes for their favorite candidate. The candidate with the fewest votes is eliminated. Anyone who voted for that eliminated candidate changes their vote to their favorite candidate who has not yet been eliminated. The winner is the candidate who is not eliminated at the end of the process.

What is the count of the votes in the first round, and which candidate is eliminated? What is the count of the votes in the second round, and which candidate is eliminated?

Part c Is there a Condorcet winner? If so, who?