UPenn NETS 412: Algorithmic Game Theory
Game Theory Practice

Problem 1

		Clyde	
		Silent	Confess
Bonnie	Silent	$-1, -1$	$-10, 0$
	Confess	$0, -10$	$-5, -5$

This game is called 'Prisoner's Dilemma'. Bonnie and Clyde have been arrested on suspicion of bank robbery, but the police do not have enough evidence to convict them without a confession. Bonnie and Clyde are separately brought to different interview rooms and asked whether or not they want to sign a confession.

The police have offered them a deal, where if one confesses and the other does not, the one confessing can walk away with no time in prison and the other will serve a sentence of 10 years. If they both confess, they can split the sentence and each serve 5 years. However, Bonnie and Clyde know that the police don't have enough evidence without a confession, so if they both stay silent, they'll each only serve one year in prison on minor charges.

Part a If Bonnie and Clyde weren't being interviewed separately, and could work together, which outcome would they pick?

Part b Suppose that Clyde has already been interviewed and has stayed silent, and Bonnie knows this. What should Bonnie do to maximize her own utility? Why?

Part c Now suppose that Clyde has already been interviewed and has confessed, and Bonnie knows this. What should Bonnie do to maximize her own utility? Why?

Part d Does Bonnie have a dominant strategy here?

Part e Does Clyde have a dominant strategy here?

Part f What is the Nash equilibrium of this game?

Part g Why aren't there any other Nash equilibria?

Problem 2

		Bob	
		Heads	Tails
Alice	Heads	$1, -1$	$-1, 1$
	Tails	$-1, 1$	$1, -1$

This game is called 'Matching Pennies' (sometimes 'Odds-and-Evens'). Alice and Bob each have a penny, and they can choose to either show heads or tails. If they both show the same thing (the pennies 'match'), Alice wins. If they show different things, Bob wins.

Part a Suppose Alice knows that Bob is going to show heads. What should she do?

Part b Suppose Alice knows that Bob is going to show tails. What should she do?

Part c Suppose Bob knows that Alice is going to show heads. What should he do?

Part d Suppose Bob knows that Alice is going to show tails. What should he do?

Part e Does either player have a strictly dominant strategy?

Part f Is there a pure strategy Nash equilibrium?

Part g Suppose instead of playing a strategy deterministically, each player randomizes. Suppose that Alice plays heads with probability p and Bob plays heads with probability q. Suppose that Alice knows that Bob has set $q = .75$. What should she set p to to maximize her utility. In other words, what is Alice's best-response to Bob's choice of $q = .75$?

Part h Suppose Alice knows that Bob has picked $q = .5$. What should she set p to to maximize her utility. In other words, what is Alice's best-response to Bob's choice of $q = .5$?

Part i If Bob has set $q = .5$, why does Alice picking $p = .9$ not result in the game being in a Nash equilibrium?

Part j Why is Alice picking $p = .5$ and Bob picking $q = .5$ a Nash equilibrium?

Part k How do we know that this Nash equilibrium is unique?

Problem 3

Both games we've seen have had a unique equilibrium, but equilibria do not have to be unique. Here we'll analyze a game with both pure and mixed strategy equilibria.

		Bob	
		Party	Movie
Alice	Party	$3, 2$	$1, 1$
	Movie	$0, 0$	$2, 3$

This game is traditionally referred to as 'Battle of the Sexes', but we'll call it 'Coordinating Activities'. Alice and Bob previously discussed going to either their friend's party or to see a movie, but neither can remember what they agreed on. Alice would rather go to the party than to the movie, and Bob would rather see the movie than go to the party, but they'd both rather do the same activity than end up at separate places.

Part a Explain why both players going to the movie or both players going to the party are both pure strategy equilibria.

Part b Explain why both players randomizing over the two choices, each choosing movie with probability .5 is not a mixed strategy Nash equilibrium.

Part c Explain why each player randomizing by picking their preferred activity with probability .75 is a mixed strategy Nash equilibrium.

Part d Are there any other mixed strategy Nash equilibria?

Problem 4

Here's a game from class.

	Violin	Harmonica
Guitar	(3,-3)	(-1,1)
Drums	(-2,2)	(1,-1)

Part a What is the column player's best response to Guitar? to Drums?

Part b What is the column player's best response if the row player is playing $p = (0.5, 0.5)$, that is, probability 0.5 on Guitar and 0.5 on Drums?

Part c This game only has one mixed Nash equilibrium. Find it!

Hint: Remember that if a player is randomizing in equilibrium, then she is indifferent between her actions, i.e. they have the same payoff.