Low-Cost Learning via Active Data Procurement

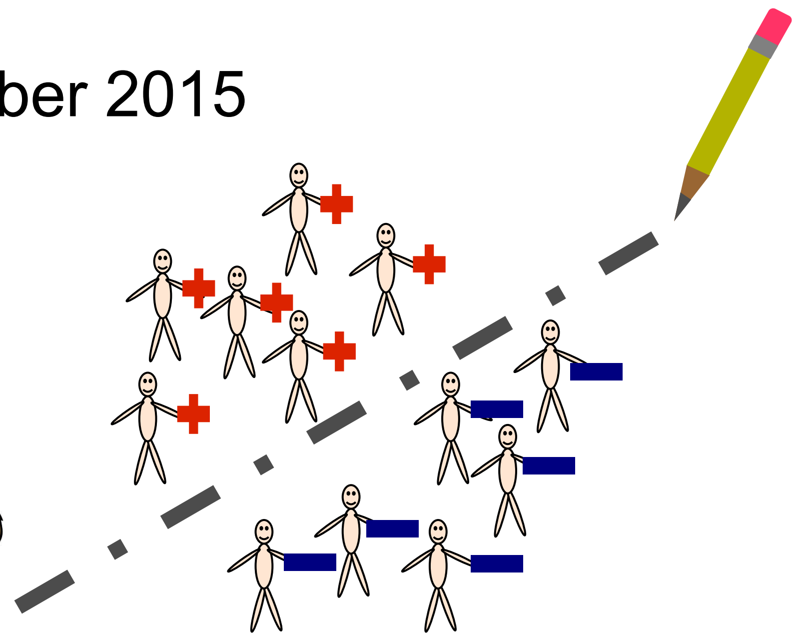
September 2015

Jacob Abernethy

Yiling Chen

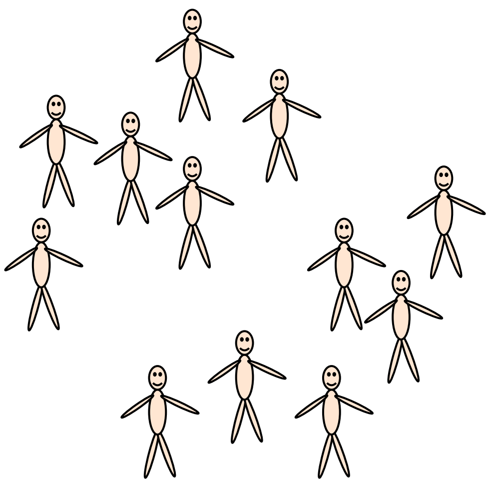
Chien-Ju Ho

Bo Waggoner



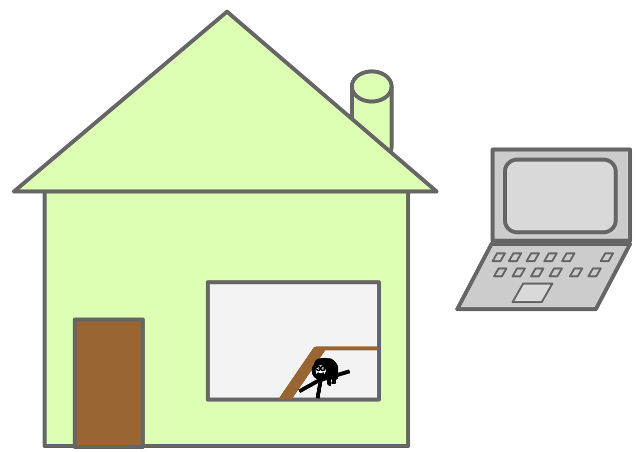
Coming soon to a society near you

data-holders



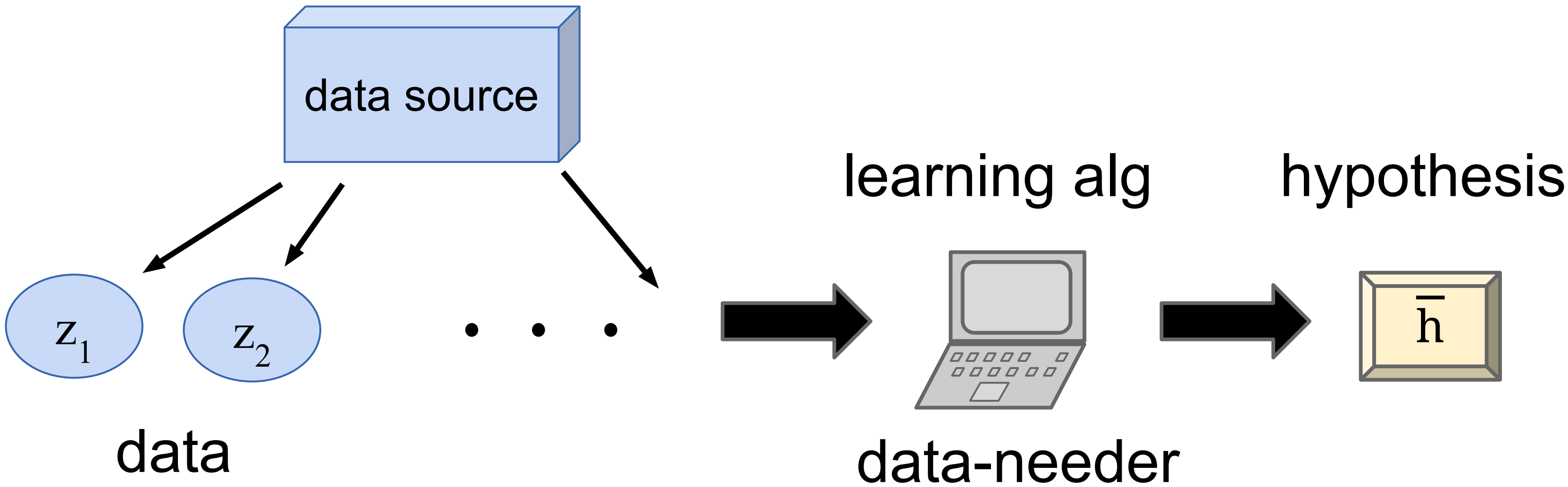
ex: medical data

data-needeers



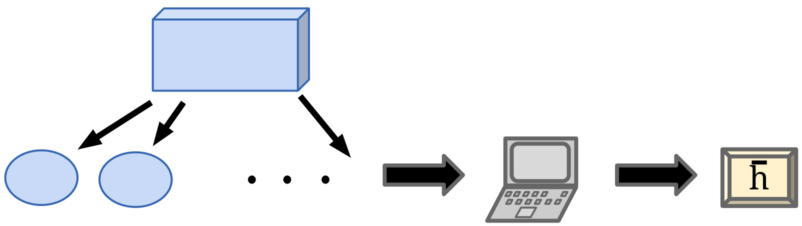
ex: pharmaceutical co.

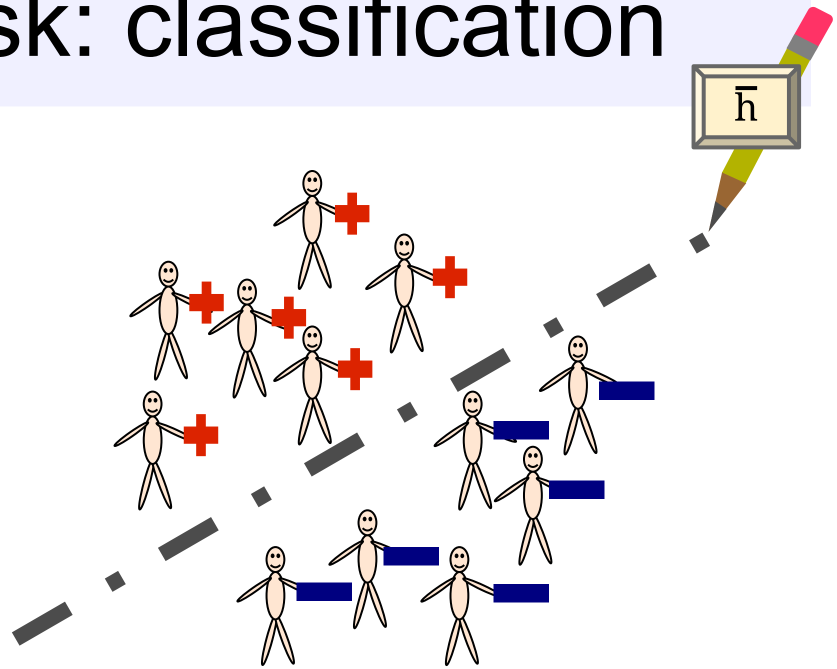
Classic ML problem



Goal: use small amount of data, output "good" h.

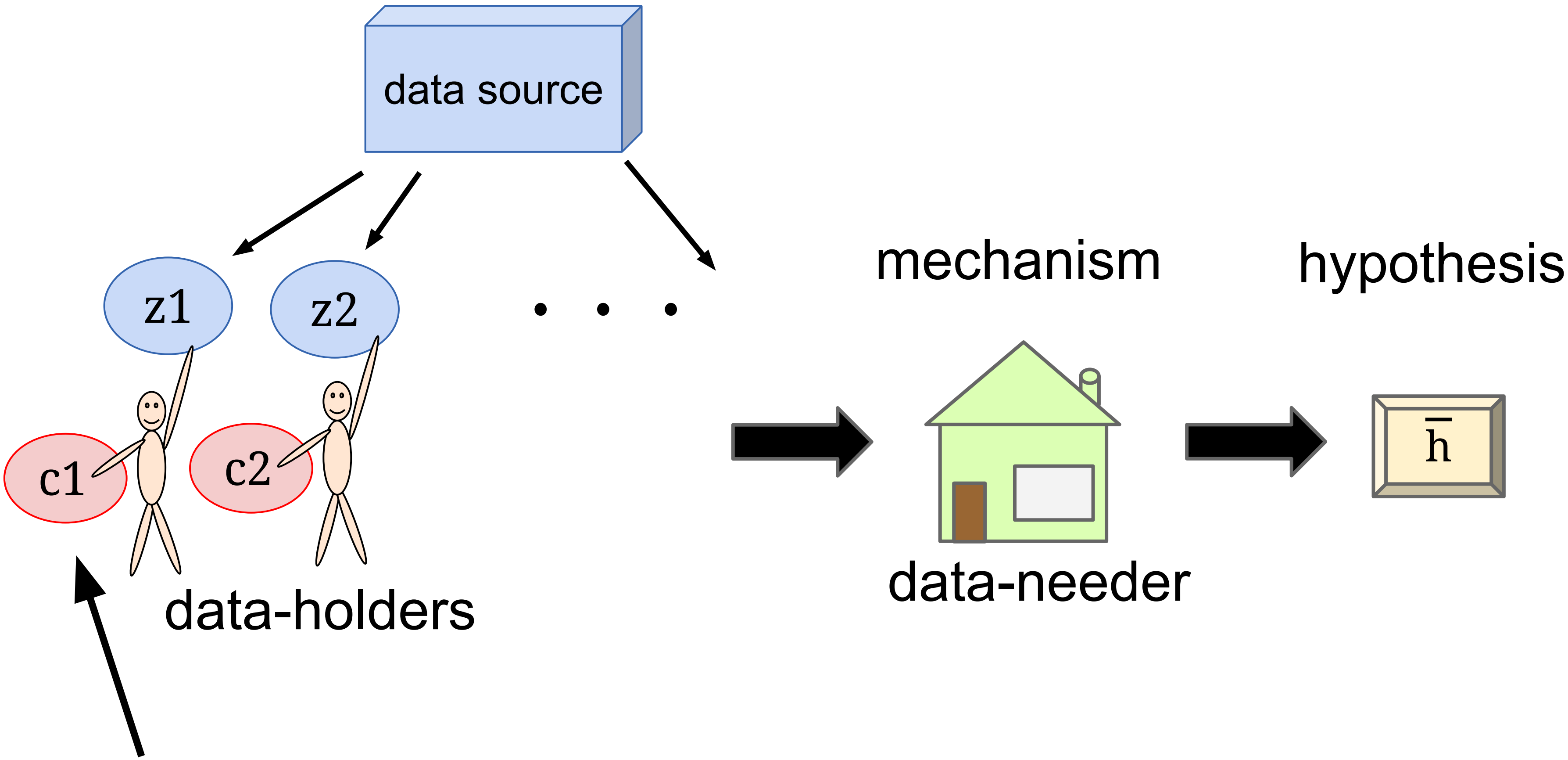
Example learning task: classification





- **Data**: (point, label) where label is + or −
- **Hypothesis**: hyperplane separating the two types

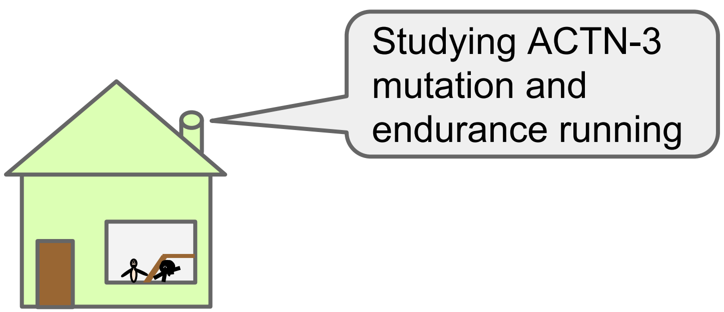
Twist: data is now held by individuals

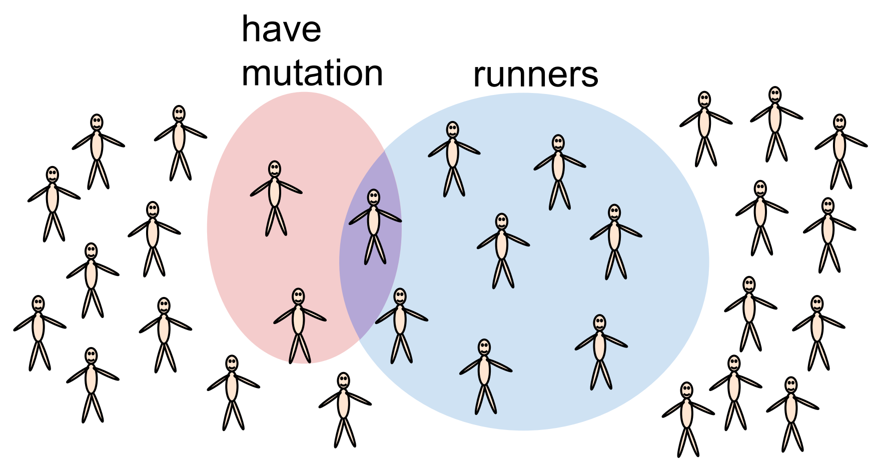


"Cost of revealing data" (formal model later…)
Goal: spend small budget, output "good" h.

Why is this difficult?

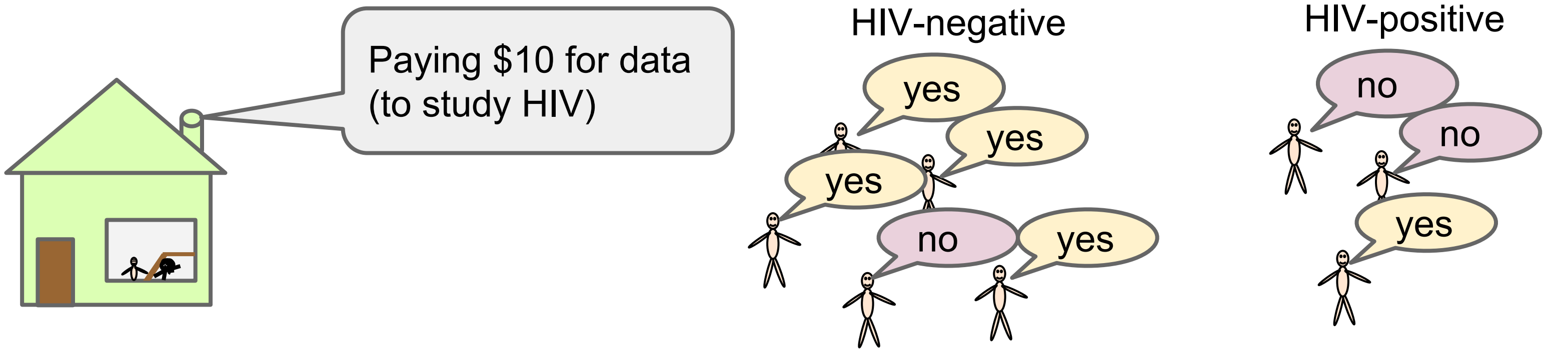
1. (Relatively) few data are **useful**





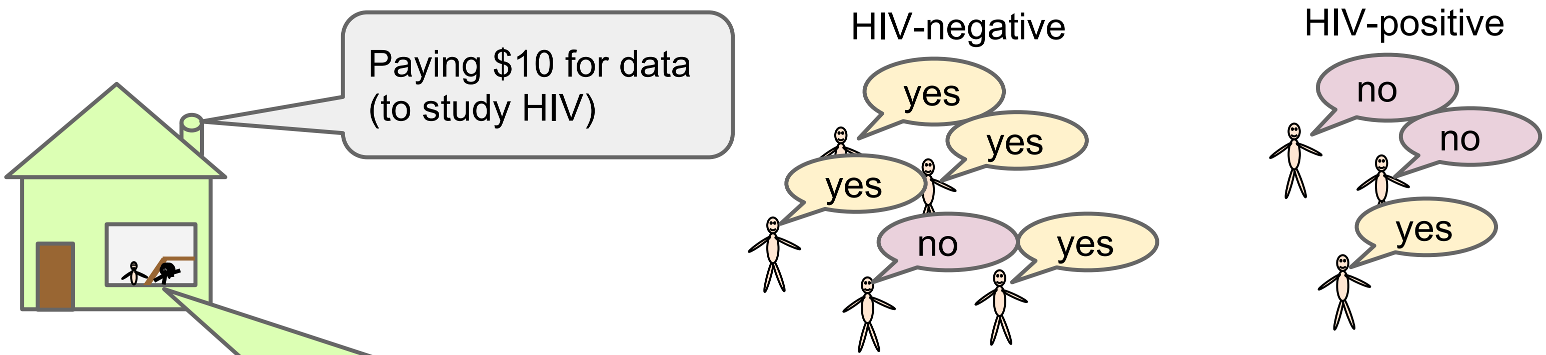
Why is this difficult?

2. Utility may be **correlated** with cost (causing bias)



Why is this difficult?

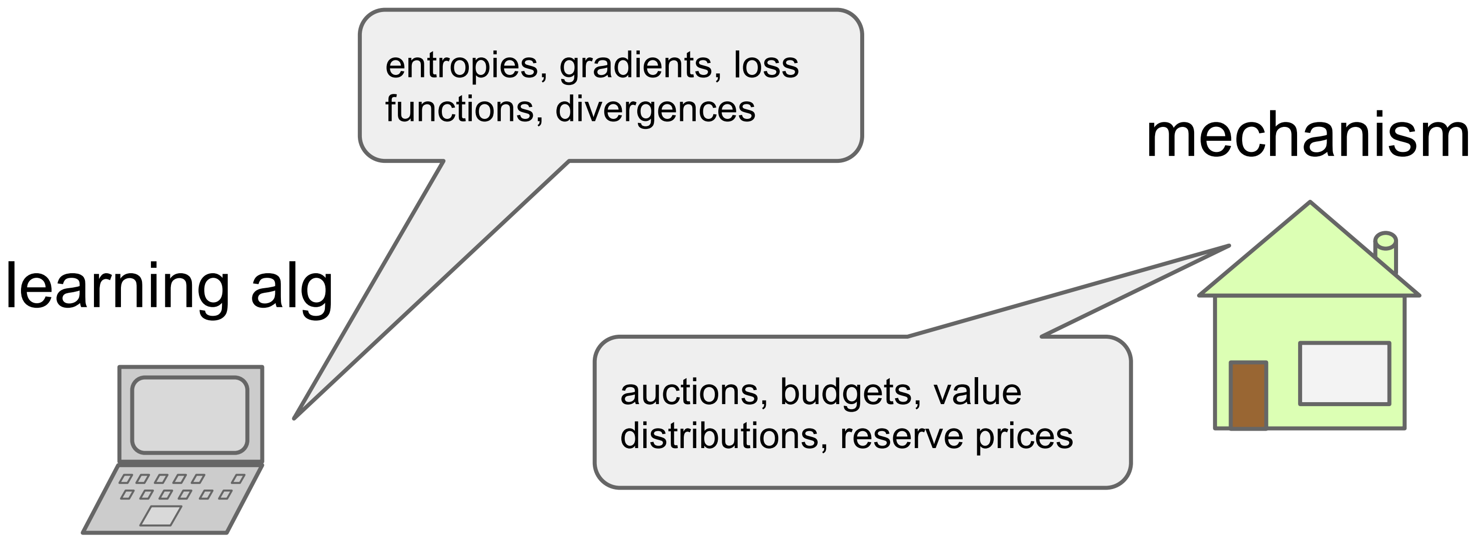
2. Utility may be **correlated** with cost (causing bias)



Machine Learning roadblock:
how to deal with biases?

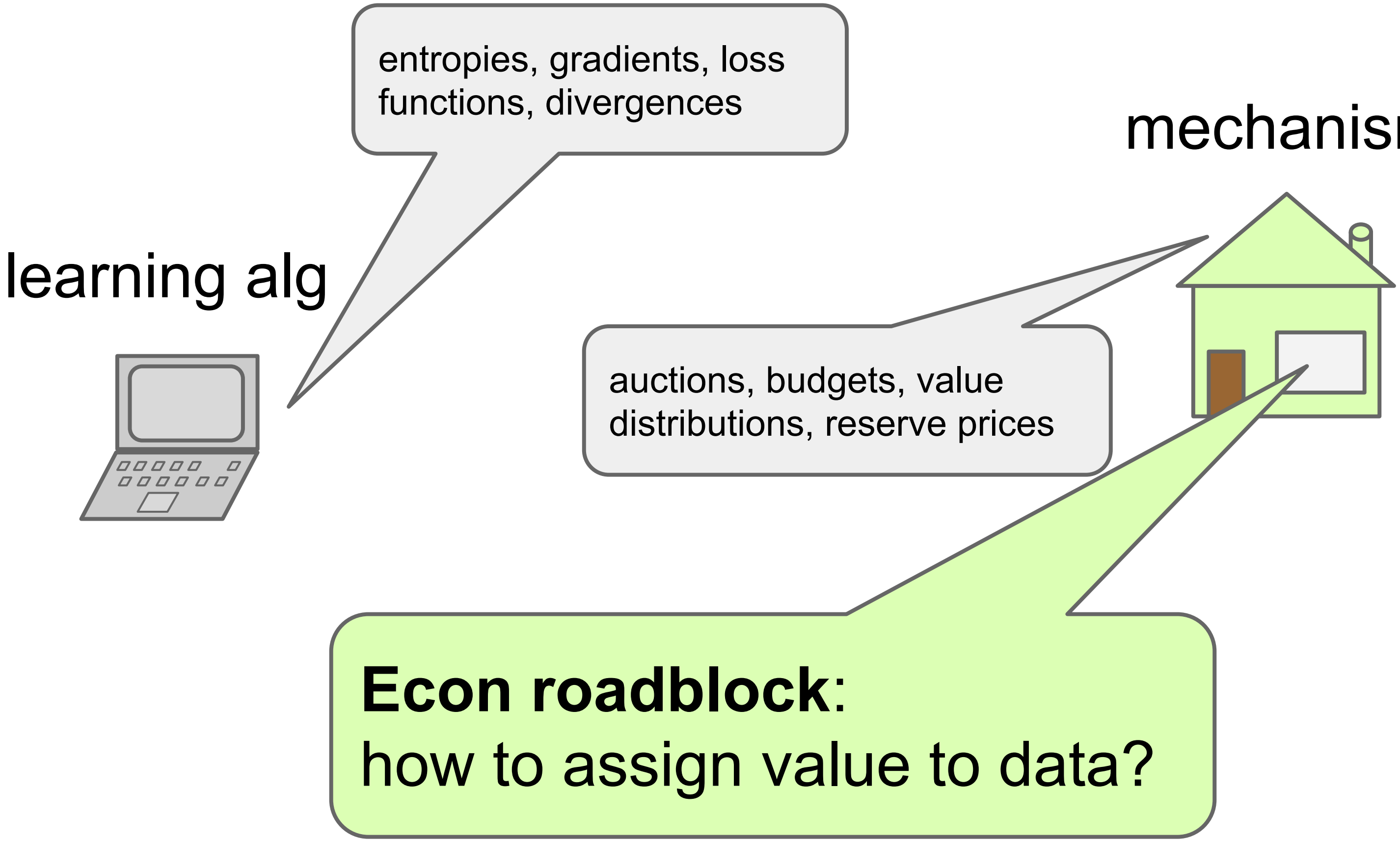
Why is this difficult?

3. Utility (ML) and cost (econ) live in **different worlds**



Why is this difficult?

3. Utility (ML) and cost (econ) live in **different worlds**



Broad research challenge:

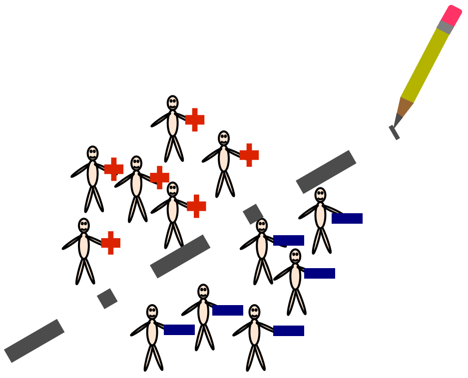
1. How to assign **value** (prices) to pieces of data?

2. How to design **mechanisms** for procuring and learning from data?

3. Develop a **theory** of budget-constrained learning: what is (im)possible to learn given budget B and parameters of the problem?

Outline

1. Overview of literature, our contributions



2. Online learning model/results

3. “Statistical learning” result, conclusion

Related work

Model: how are agents strategic?

agents cannot fabricate data, have costs

this work

Roth, Schoenebeck 2012

Ligett, Roth 2012

Horel, Ionnadis, Muthukrishnan 2014

principal-agent style, data depends on effort

Cummings, Ligett, Roth, Wu, Ziani 2015

Cai, Daskalakis, Papadimitriou 2015

can fabricate data (like in peer-prediction)

Meir, Procaccia, Rosenschein 2012

Dekel, Fisher, Procaccia 2008

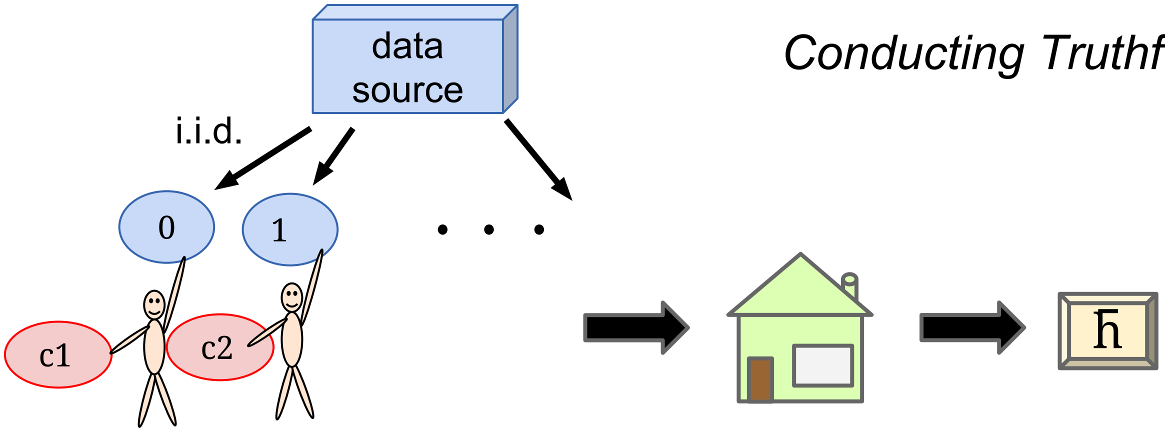
Ghosh, Ligett, Roth, Schoenebeck 2014

Related work

Type of goal →	risk/regret bounds	minimize variance or related goal
agents cannot fabricate data, have costs	this work	Roth, Schoenebeck 2012 Ligett, Roth 2012 Horel, Ionnadis, Muthukrishnan 2014
principal-agent style, data depends on effort		Cummings, Ligett, Roth, Wu, Ziani 2015 Cai, Daskalakis, Papadimitriou 2015
can fabricate data (like in peer-prediction)	Meir, Procaccia, Rosenschein 2012	Dekel, Fisher, Procaccia 2008 Ghosh, Ligett, Roth, Schoenebeck 2014

e.g. Roth-Schoenebeck, EC 2012

Conducting Truthful Surveys, Cheaply



- Each datapoint is a number. Task is to **estimate the mean**
- **Approach:** offer each agent a price drawn i.i.d.
- **Idea:** obtains cheap but biased data; can de-bias it
- **Result:** derives *price distribution* to minimize variance of estimate

What we wanted to do differently

1. **Prove ML-style risk or regret bounds**
   rather than “minimize the variance” type goals.
   Why: understand error rate as function of budget and problem characteristics (as in ML)

What we wanted to do differently

1. **Prove ML-style risk or regret bounds**
   rather than "minimize the variance" type goals.
   Why: understand error rate as function of budget and problem characteristics (as in ML)

2. **Interface with existing ML algorithms.**
   Why: understand how value derives from learning alg.
   Toward black-box use of learners in mechanisms.

“general” learning problems

population average

regression

classification

	risk bounds	minimize or related goal
agents cannot fabricate data, have costs	**this work**	Roth, Schoenebeck 2012 Ligett, Roth 2012 Horel, Ionnadis, Muthukrishnan 2014
principal-agent style, data depends on effort		Cummings, Ligett, Roth, Wu, Ziani 2015 Cai, Daskalakis, Papadimitriou 2015
can fabricate data (like in peer-prediction)	Meir, Procaccia, Rosenschein 2012	Dekel, Fisher, Procaccia 2008 Ghosh, Ligett, Roth, Schoenebeck 2014

What we wanted to do differently

1. **Prove ML-style risk or regret bounds**
   rather than "minimize the variance" style.
   Why: understand error rate as function of budget and problem characteristics (as in ML)

2. **Interface with existing ML algorithms.**
   Why: understand how value derives from learning alg. Toward black-box use of learners in mechanisms.

3. **Online data arrival**
   rather than "batch" setting.
   Why: allows "active learning" approach, nice model

Related work

online, active

“batch”

	risk/regret bounds	minimize variance or related goal
agents cannot fabricate data, have costs	this work	Roth, Schoenebeck 2012 Ligett, Roth 2012 Horel, Ionnadis, Muthukrishnan 2014
principal-agent style, data depends on effort		Cummings, Ligett, Roth, Wu, Ziani 2015 Cai, Daskalakis, Papadimitriou 2015
can fabricate data (like in peer-prediction)	Meir, Procaccia, Rosenschein 2012	Dekel, Fisher, Procaccia 2008 Ghosh, Ligett, Roth, Schoenebeck 2014

Related work

	risk/regret bounds	**minimize variance or related goal**
agents cannot fabricate data, have costs	**this work**	Roth, Schoenebeck 2012 Ligett, Roth 2012 Horel, Ionnadis, Muthukrishnan 2014
principal-agent style, data depends on effort		Cummings, Ligett, Roth, Wu, Ziani 2015 Cai, Daskalakis, Papadimitriou 2015
can fabricate data (like in predict	Meir, Procaccia	Dekel, Fisher, Procaccia 2008 Ghosh, Ligett, Roth, Schoenebeck 2014

Abernethy, Frongillo, W. NIPS 2015

Overview of our contributions

Propose model of online learning with purchased data: T arriving data points and budget B.

Convert any “FTRL” algorithm into a mechanism.

Show regret on order of $T / \sqrt{B}$
and lower bounds of same order.

Overview of our contributions

Extend model to case where data is drawn i.i.d. (“statistical learning”)

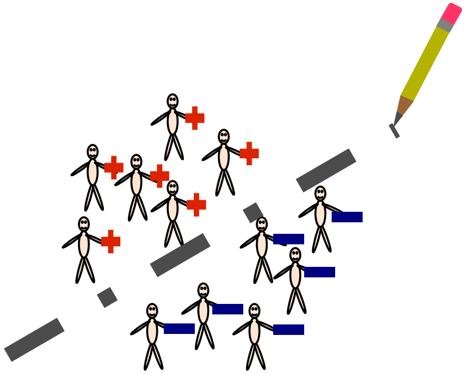
Propose model of online learning with purchased data: T arriving data points and budget B.

Convert any “FTRL” algorithm into a mechanism.

Show regret on order of $T / \sqrt{B}$
and lower bounds of same order.

Extend result to “risk” bound on order of $1 / \sqrt{B}$.

Outline



1. Overview of literature, our contributions

2. Online learning model/results

3. “Statistical learning” result, conclusion

Online learning with purchased data

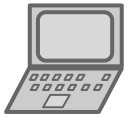
a. Review of online learning

b. Our model: adding $$

c. Deriving our mechanism and results

Standard online learning model

For $t = 1, \ldots, T$:



- algorithm posts a hypothesis h_t
- data point z_t arrives
- algorithm sees z_t and updates to h_{t+1}

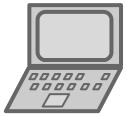
Loss **=** $\sum_t \ell(h_t, z_t)$

Regret **=** **Loss** - $\sum_t \ell(h^*, z_t)$ where h^* minimizes sum

Follow-the-Regularized-Leader (FTRL)

Assume: loss function is convex and Lipschitz, hypothesis space is Hilbert, etc

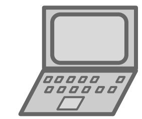
Algorithm: $h_t = \mathrm{argmin} \sum_{s<t} \ell(h, z_s) + R(h)/\eta$



Follow-the-Regularized-Leader (FTRL)

Assume: loss function is convex and Lipschitz, hypothesis space is Hilbert, etc

Algorithm: $h_t = \mathrm{argmin} \sum_{s<t} \ell(h, z_s) + R(h)/\eta$



Example 1 (Euclidean norm): $R(h) = \|h\|_2^2$

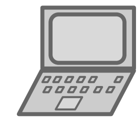
$\Rightarrow h_t = h_{t-1} - \eta \nabla \ell(h, z_t)$

online gradient descent

Follow-the-Regularized-Leader (FTRL)

Assume: loss function is convex and Lipschitz, hypothesis space is Hilbert, etc

Algorithm: $h_t = \operatorname{argmin} \sum_{s<t} \ell(h, z_s) + R(h)/\eta$



Example 1 (Euclidean norm): $R(h) = \|h\|_2^2$

$\Rightarrow h_t = h_{t-1} - \eta \nabla \ell(h, z_t)$

online gradient descent

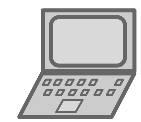
Example 2 (negative entropy): $R(h) = \sum_j h^{(j)} \ln(h^{(j)})$.

$\Rightarrow h_t^{(j)} \propto h_{t-1}^{(j)} \exp[\, \eta \nabla \ell(h_{t-1}, z_t)\,]$

multiplicative weights

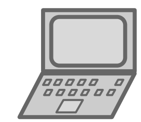
Regret Bound for FTRL

Fact: the regret of FTRL is bounded by O of
$1/\eta + \eta \sum_t \Delta_t^2$ where $\Delta_t = \| \nabla \ell(h_t, z_t) \|$.



Regret Bound for FTRL

Fact: the regret of FTRL is bounded by O of
$1/\eta + \eta \sum_t \Delta_t^{2}$ where $\Delta_t = \| \nabla \ell(h_t, z_t) \|$.



We know $\Delta_t \leq 1$ by assumption, so we can choose $\eta = 1/\sqrt{T}$ and get Regret $\leq O(\sqrt{T})$.

"No regret": average regret $\to 0$.

Online learning with purchased data

a. Review of online learning

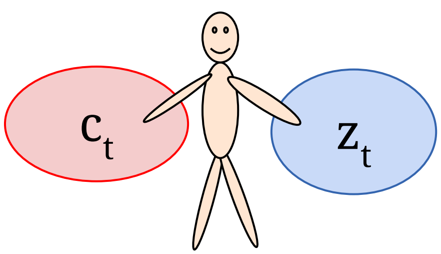
b. Our model: adding $$

c. Deriving our mechanism and results

Model of strategic data-holder

Model of agent:

- holds data z_t and cost c_t
- cost is **threshold price**
  - agent agrees to sell data iff price $\geq c_t$
  - interpretations: privacy, transaction cost, ….



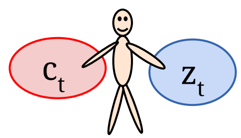
- Assume: all costs ≤ 1

Model of agent-mechanism interaction

- Mechanism posts **menu** of prices offered:

data:	(32,12) −	(20,18) +	(32,12) +
price:	\$0.22	\$0.41	\$0.88

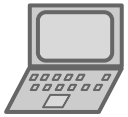
- agent t arrives



- If $c_t \leq \mathrm{price}(z_t)$, agent **accepts**:
  - agent reveals (z_t, c_t)
  - mechanism pays agent $\mathrm{price}(z_t)$
- Otherwise, agent **rejects**:
  - mechanism learns that agent rejected, pays nothing

Recall: standard online learning model

For $t = 1, \ldots, T$:

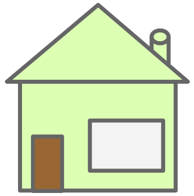


- algorithm posts a hypothesis h_t
- data point z_t arrives
- algorithm sees z_t and updates to h_{t+1}

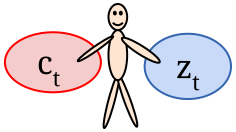
Our model: online learning with $$

For $t = 1, \ldots, T$:

- mechanism posts a hypothesis h_t *and* a menu of prices



- data point z_t arrives with cost c_t



- If $c_t \leq$ *menu price* of z_t: mech pays price, learns z_t
- else: mech pays nothing

Loss **=** $\sum_t \ell(h_t, z_t)$
Regret **=** **Loss** - $\sum_t \ell(h^*, z_t)$ where h^* minimizes sum

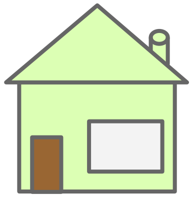
Online learning with purchased data

a. Review of online learning

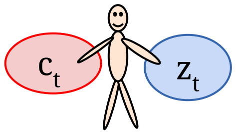
b. Our model: adding $$

c. Deriving our mechanism and results

Start easy



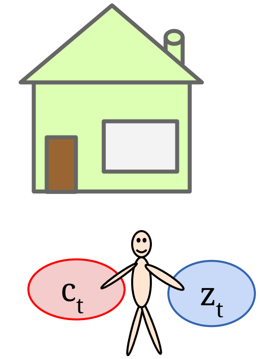
Suppose all costs are 1.
⇒ Determine which data points to sample.



data:	(32,12) −	(20,18) +	(32,12) +
price:	\$1	\$0	\$0

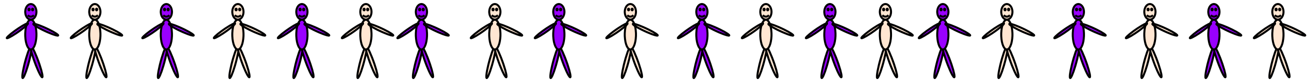
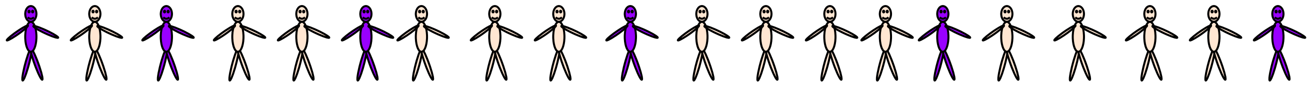
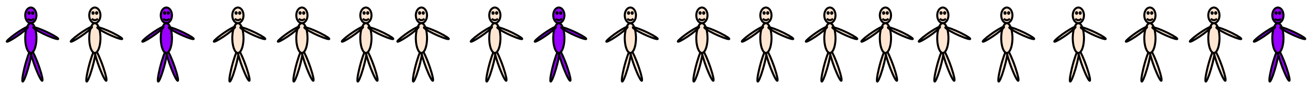
Start easy

Suppose all costs are 1.
⇒ Determine which data points to sample.



data:	(32,12) −	(20,18) +	(32,12) +
price:	\$1	\$0	\$0

Examples:

- $B = T/2$ 
- $B = \sqrt{T}$ 
- $B = \log(T)$ 

Key idea #1: randomly sample

Can purchase each data point z_t with probability $q_t(z_t)$.

Menu is now **randomly chosen**:

data:	(32,12) −	(20,18) +	(32,12) +
Pr[price=1]:	0.3	0.06	0.41

Key idea #1: randomly sample

Can purchase each data point z_t with probability $q_t(z_t)$.

Menu is now **randomly chosen**:

data:	(32,12) $-$	(20,18) $+$	(32,12) $+$
Pr[price=1]:	0.3	0.06	0.41

Lemma (importance-weighted regret bound):
For any q_ts, the regret of (modified) FTRL is O of

$$1/\eta + \eta \, E \left[\sum_t (\Delta_t^2 / q_t) \right]$$

See also: *Importance-Weighted Active Learning*, Beygelzimer et al, ICML 2009.

Result for easy case

Lemma (importance-weighted regret bound):
For any q_ts, the regret of (modified) FTRL is O of

$$1/\eta \ + \eta \, E \left[\textstyle\sum_t (\Delta_t^{\,2} / q_t) \right]$$

Corollary:
Setting all q_t = B/T and choosing $\eta = \sqrt{B} / T$ yields

regret $\leq T / \sqrt{B}$.

"No data, no regret":
average amount of data → 0 *and* average regret → 0.

Result for easy case

Lemma (importance-weighted regret bound):
For any q_ts, the regret of (modified) FTRL is O of

$$1/\eta \; + \eta \, E \left[\sum_t (\Delta_t^{\,2} / q_t) \right]$$

Corollary:
Setting all q_t = B/T and choosing $\eta = \sqrt{B} / T$ yields
regret $\leq T / \sqrt{B}$.

Theorem:
This is tight.

(Predict a repeated coin toss whose bias is either $1+1/\sqrt{B}$ or $1-1/\sqrt{B}$)

Now a bit harder….

Costs can be arbitrary, but agents are **nonstrategic**: they will accept payment exactly c_t.

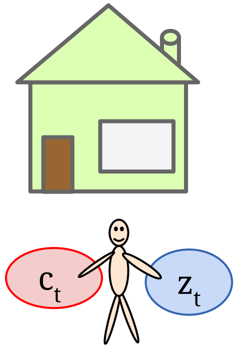
At each time step, randomly choose which (data, cost) pairs to purchase.

data,cost:	(32,12), c=0.3	(20,18), c=0.8
Pr[purchase]:	0.12	0.08

Question: how to set probabilities of purchase q_t?

Key idea #2: sample proportional to...

Imagine we knew the arrivals in advance.
Optimization problem:

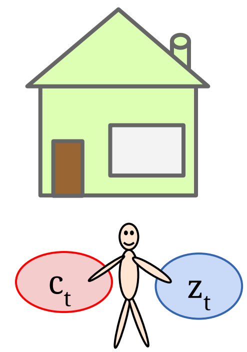


$$
\boxed{
\begin{array}{lll}
\text{minimize} & \sum_t (\Delta_t^2 / q_t) & \\
 & & \\
\text{s.t.} & \sum_t q_t c_t & \leq B \\
 & q_t & \leq 1.
\end{array}
}
$$

Solution: $q_t = \Delta_t / K \sqrt{c_t}$ (K a normalizing constant).

Key idea #2: sample proportional to...

Imagine we knew the arrivals in advance.
Optimization problem:



$$\begin{array}{lll} \text{minimize} & \sum_t (\Delta_t^2 / q_t) & \\ \text{s.t.} & \sum_t q_t \, c_t & \leq B \\ & q_t & \leq 1. \end{array}$$

Solution: $q_t = \Delta_t \,/\, K \sqrt{c_t}$ (K a normalizing constant).

The point: only need advance knowledge of K to implement the “optimal” sampling strategy!

Turns out: K = $\gamma\, T / B$, where $\gamma \in [0,1]$ (discuss later)

Result for this "at-cost" setting

Theorem:
Given rough advance estimate of γ, can achieve

$$\text{regret} \leq \gamma T / \sqrt{B}$$

Theorem:
This is tight (in a reasonable sense).

(Same bad instance, but with "useless" free data points sprinkled in.)

Implication: γ is capturing the "difficulty of the problem".

Discussion

$\gamma = (1/T) \sum_t \Delta_t \sqrt{c_t}$
$= \text{average sqrt(difficulty * cost)}.$

Discussion

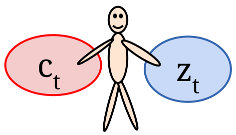
$\gamma = (1/T) \sum_t \Delta_t \sqrt{c_t}$
$= \text{average sqrt(difficulty * cost)}.$

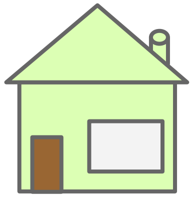
Example simplified corollary:
Given rough advance estimate of avg cost μ,
regret $\leq \sqrt{\mu}\, T / \sqrt{B}$

- Low avg cost $\Rightarrow$ low regret
- Low avg difficulty $\Rightarrow$ low regret
- **good correlations** $\Rightarrow$ low regret

Finally, the "full" problem.

Now agents are **strategic**
and we must **post prices**.



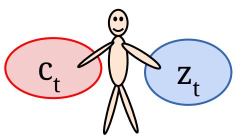


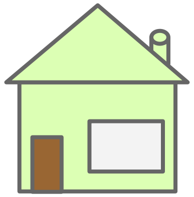
Recall: had sampling probability $q_t = \Delta_t \ / \ K \sqrt{c_t}$.

But: we don't know c_t.

Finally, the “full” problem.

Now agents are **strategic**
and we must **post prices**.





Recall: had sampling probability $q_t = \Delta_t \, / \, K \sqrt{c_t}$.

But: we don’t know c_t.

Key idea #3: randomly draw price from the distribution s.t.
$\Pr[\text{ price} \geq c_t] = \Delta_t \, / \, K \sqrt{c_t}$.

$\Rightarrow$ achieve the “right” probability for *every* c_t simultaneously!

Description of final mechanism

Input: estimate of γ

At each time t:

- post hypothesis $h_t \leftarrow$ FTRL
- for each data point z_t, compute $\Delta_t = \| \nabla \ell(h_t, z_t) \|$ and post random price from distribution
- If arriving agent accepts, send "re-weighted" $z_t \rightarrow$ FTRL

Main result for online learning setting

Theorem:
Given rough advance estimate of γ, can achieve

$$\text{regret} \leq \sqrt{\gamma}\, T / \sqrt{B}$$

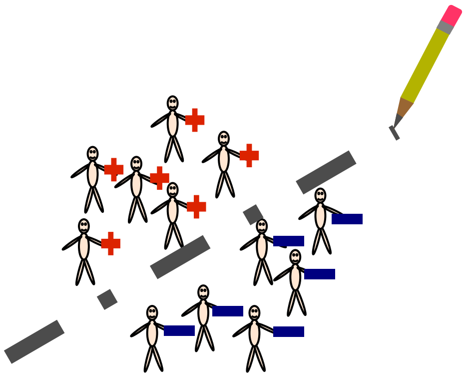
Theorem (recall):
No mechanism for the easier, “at-cost” setting can beat

$$\text{regret} \leq \gamma\, T / \sqrt{B}$$

Note: lost a $\sqrt{\gamma}$ factor compared to easier setting,
due to paying our posted price rather than the agent’s cost.
(“cost of strategic behavior”)

Outline

1. Overview of literature, our contributions



2. Online learning model/results
3. “Statistical learning” result, conclusion

Recalling contributions

Extend model to case where data is drawn i.i.d. ("statistical learning")

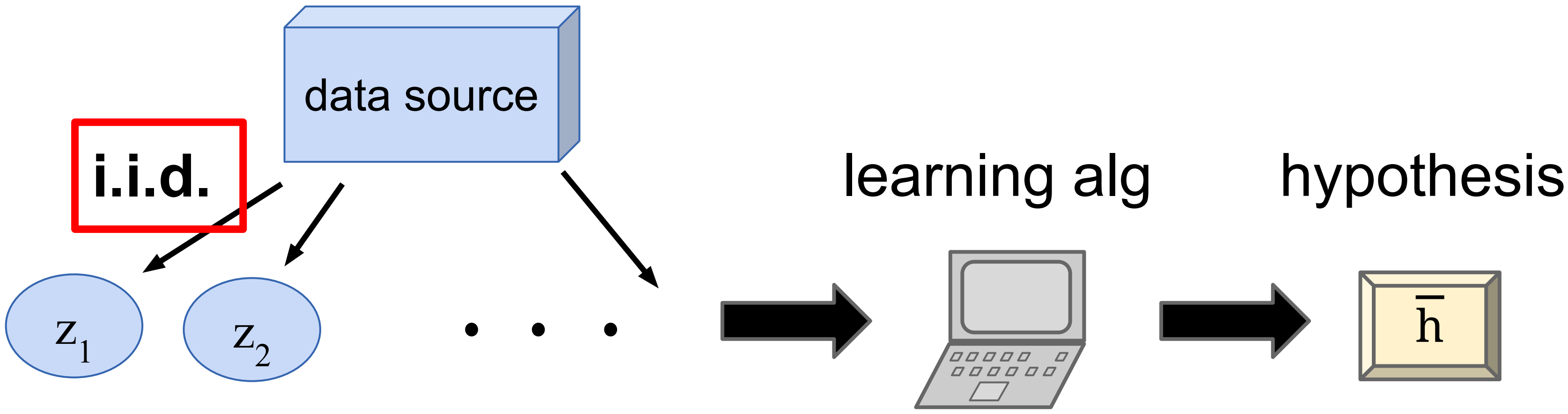
Propose model of online learning with purchased data: T arriving data points and budget B.

Convert any "FTRL" algorithm into a mechanism.

Show regret on order of $T / \sqrt{B}$
and lower bounds of same order.

Extend result to "risk" bound on order of $1 / \sqrt{B}$.

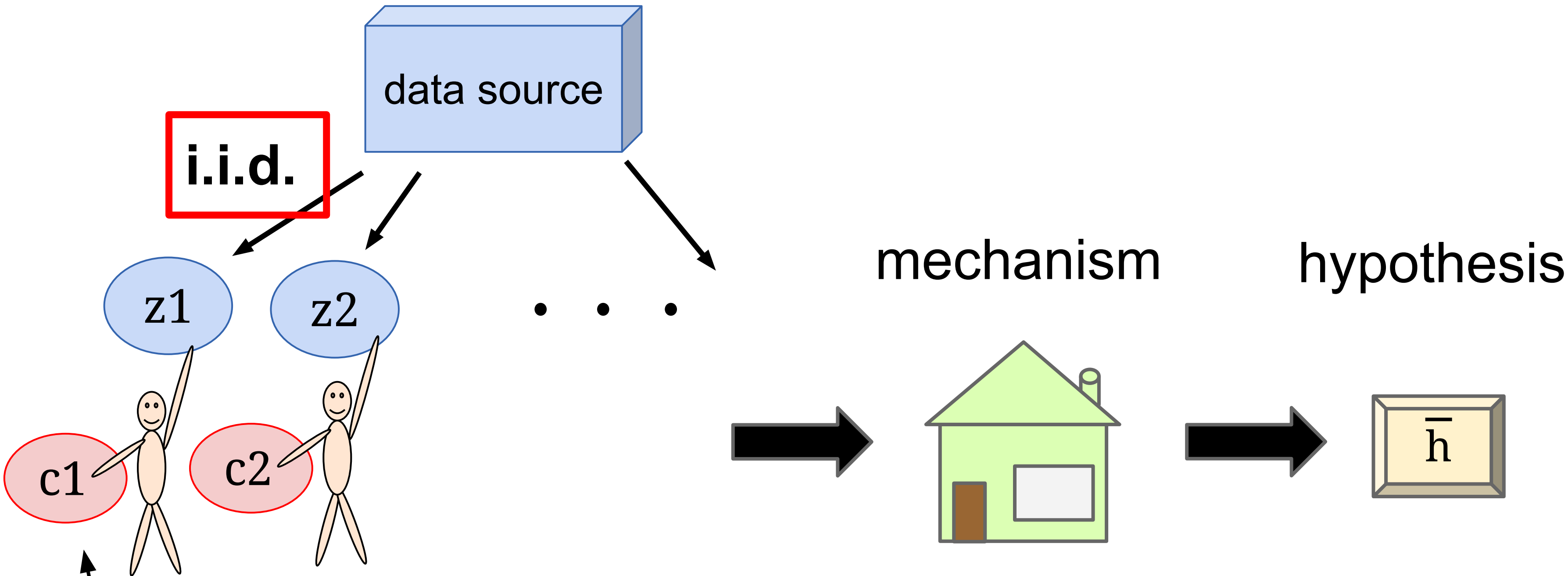
Classic statistical learning model



For classification:

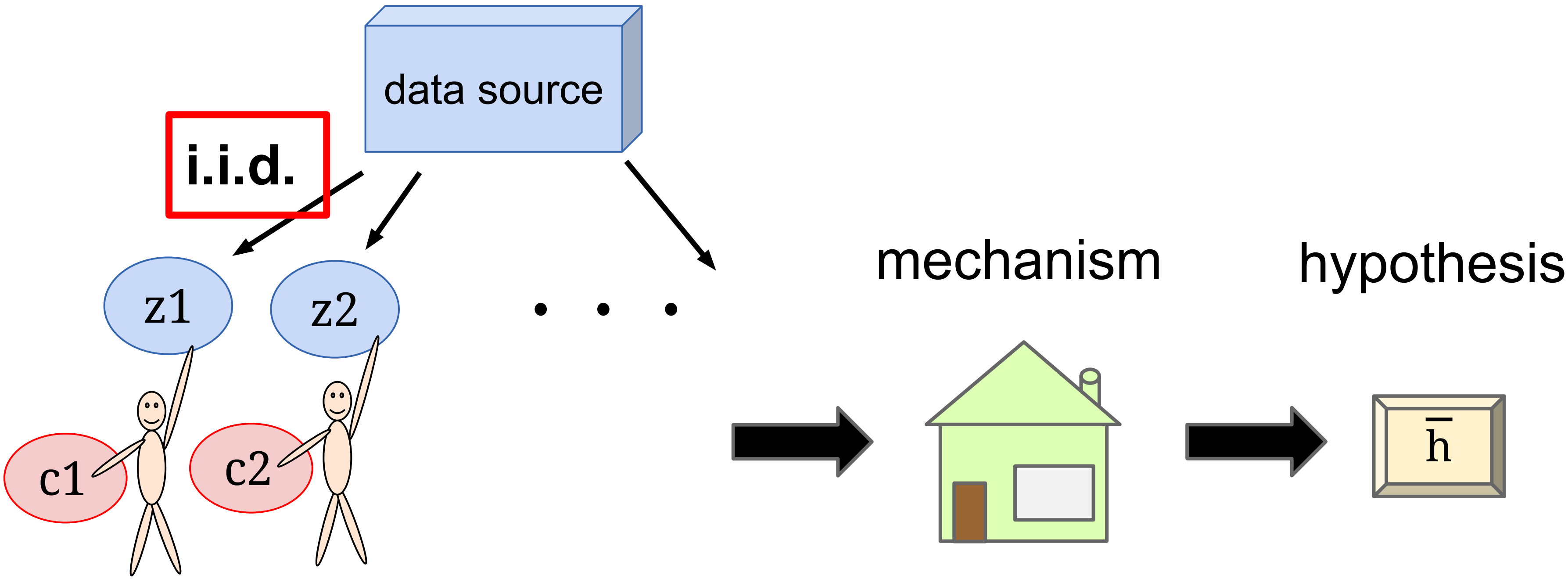
$$\mathrm{E}\,\mathrm{loss}(\,\bar{h}\,) \leq \mathrm{E}\,\mathrm{loss}(\,h^*\,) + O\left(\sqrt{\frac{\text{VC-dim}}{T}}\right)$$

Our statistical learning model



costs (still) may be adversarially chosen

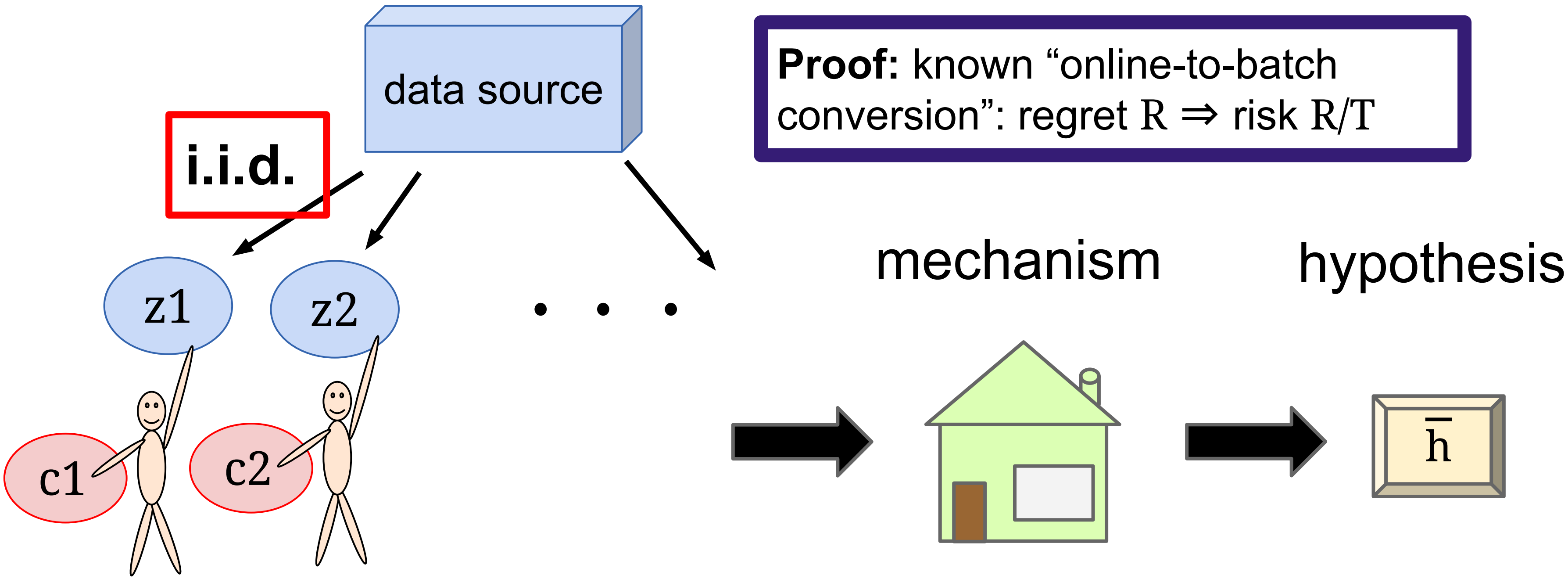
Our statistical learning model



Theorem:
Given rough advance estimate of γ, can achieve

$$\mathrm{E}\,\mathrm{loss}(\,\bar{h}\,) \leq \mathrm{E}\,\mathrm{loss}(\,h^*\,) + O\left(\sqrt{\frac{\gamma}{B}}\right)$$

Our statistical learning model

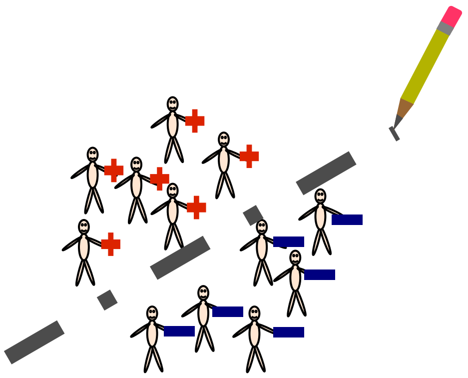


Proof: known "online-to-batch conversion": regret R ⇒ risk R/T

Theorem:
Given rough advance estimate of γ, can achieve

$$\mathrm{E}\,\mathrm{loss}(\,\bar{h}\,) \le \mathrm{E}\,\mathrm{loss}(\,h^*\,) + O\left(\sqrt{\frac{\gamma}{B}}\right)$$

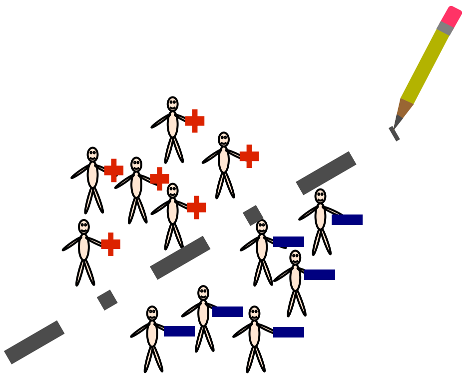
Summary



Model:

- online arrival of agents
- post prices to procure data
- adversarial costs and data (online learning setting)
- adversarial costs, i.i.d. data (statistical learning setting)

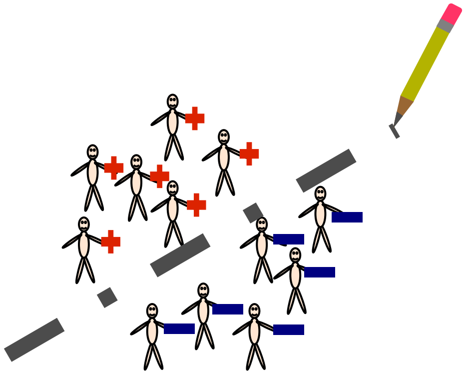
Summary



Results:

- upper/lower bounds on regret
  (online learning setting)
- upper bound on risk
  (statistical learning setting)

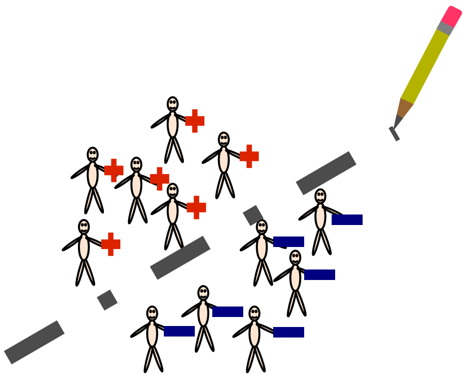
Summary



Big picture:

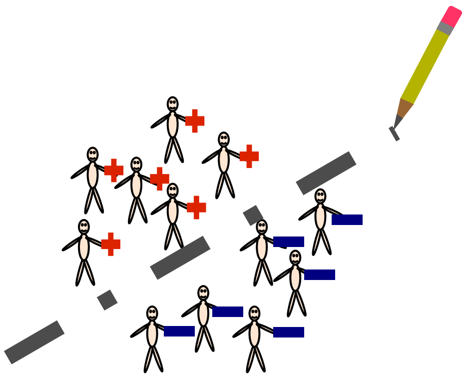
- design mechanisms to interface with existing learning algs
- prove ML-style bounds: risk and regret
- toward a “theory of the learnable...on a budget”

Future work



- Improve bounds (!)
- Propose "universal quantity" to replace γ in bounds (analogue of VC-dimension?)
- Explore models for purchasing data

Future work



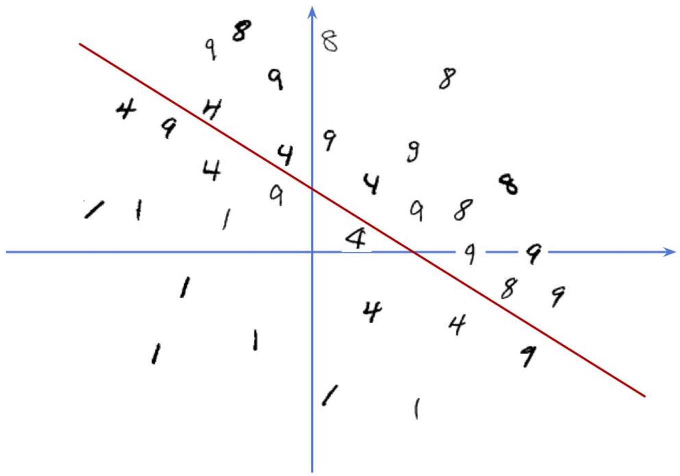
- Improve bounds (!)
- Propose "universal quantity" to replace γ in bounds (analogue of VC-dimension?)
- Explore models for purchasing data

Thanks!

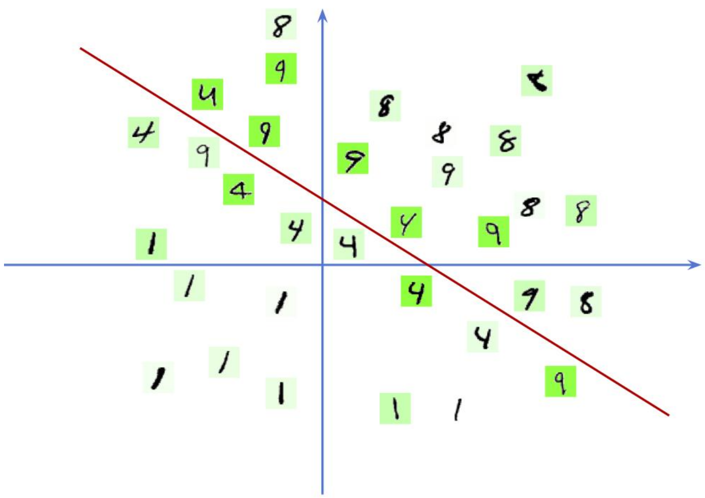
Additional slides

Simulation results

MNIST dataset -- handwritten digit classification



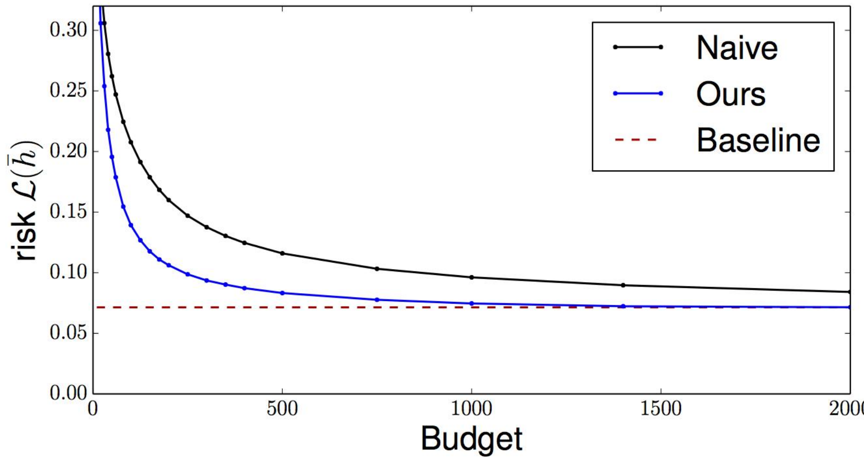
Toy problem: classify (1 or 4) vs (9 or 8)



Brighter green = higher cost

Simulation results

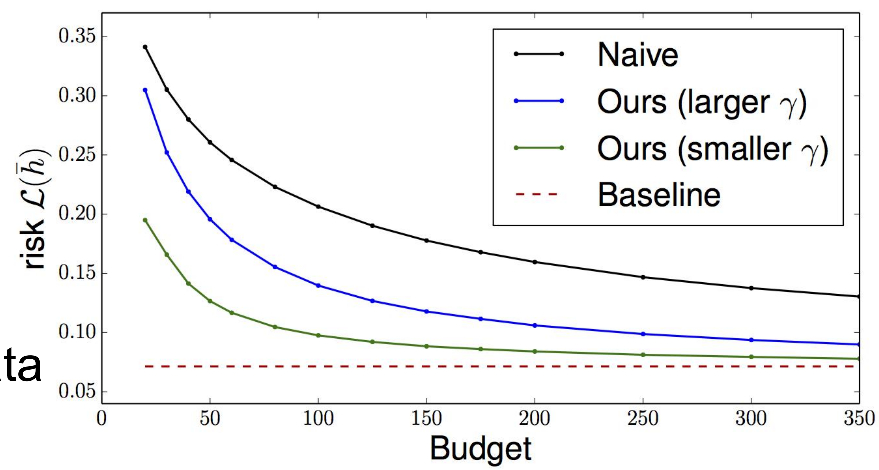
- T = 8503
- train on half, test on half
- Alg: Online Gradient Descent



Naive: pay 1 until budget is exhausted, then run alg

Baseline: run alg on all data points (no budget)

Large γ: bad correlations
Small γ: independent cost/data



Pricing distribution