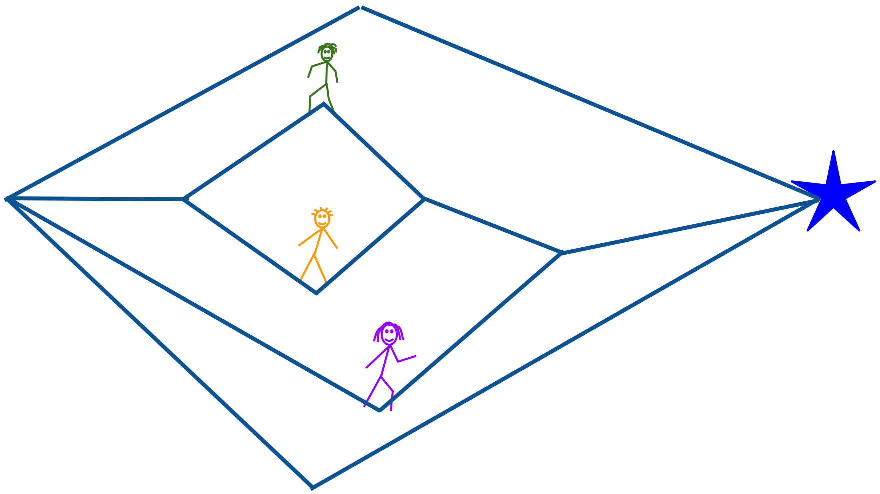
Informational Substitutes



Yiling Chen Harvard
Bo Waggoner UPenn

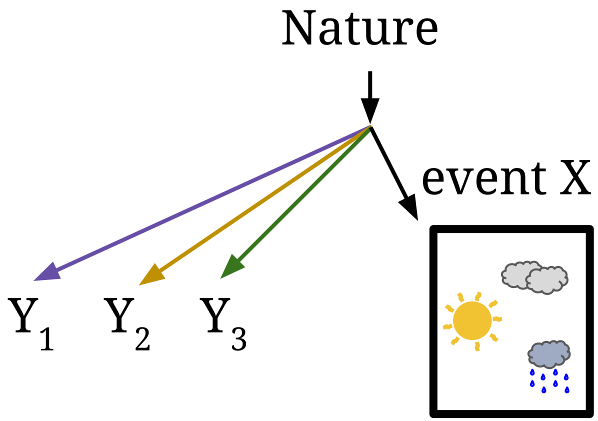
0. Information

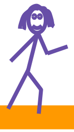
Information (in this talk)

Random variables $X, Y_1, \ldots, Y_n$ jointly distributed, known prior. (finite set of outcomes)

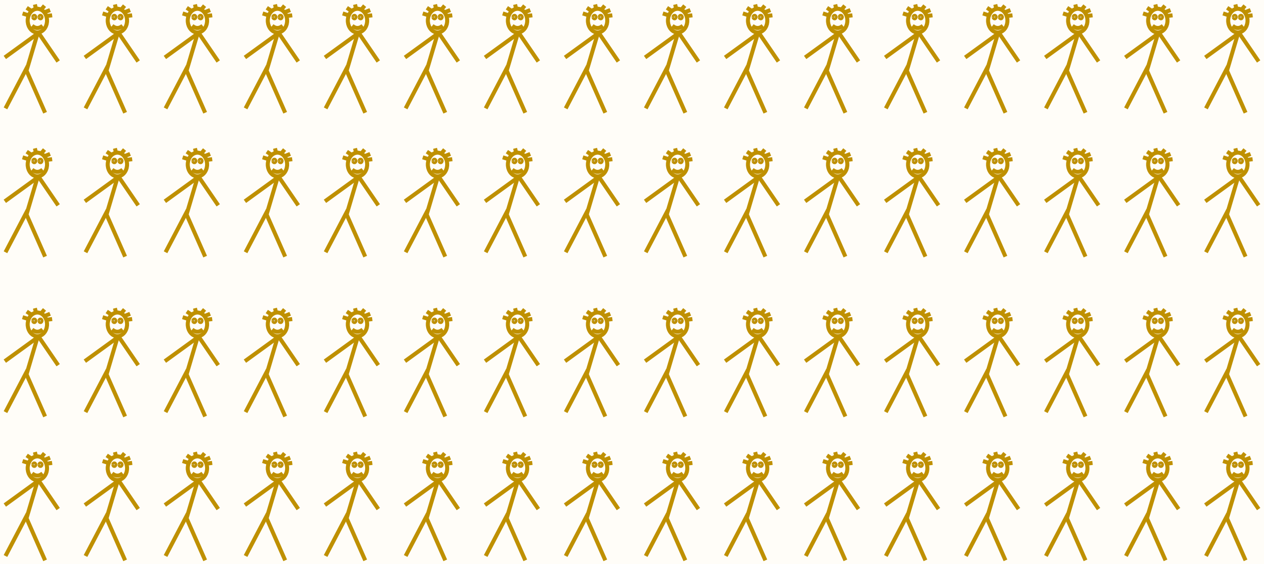
We care about X.

Y_i = "signal" (reveals info. about X).





1. Substitutes



The unreasonable effectiveness of substitutes

Substitutes in economics:

- Market equilibria, stable matchings, …
- [Kelso & Crawford 1982, Roth 1984, Hatfield and Milgrom 2005, …]

Substitutes in computer science:

- Submodularity! [Lehmann+Lehmann+Nisan 2001]
- subs == efficient approx. for **many** problems

Could we also define “substitutes” for information?

And could they also link algorithms and game theory?

Challenges for defining informational S&C

Items

- Valuation function f is given

- f(S) does not depend on f(T)

…

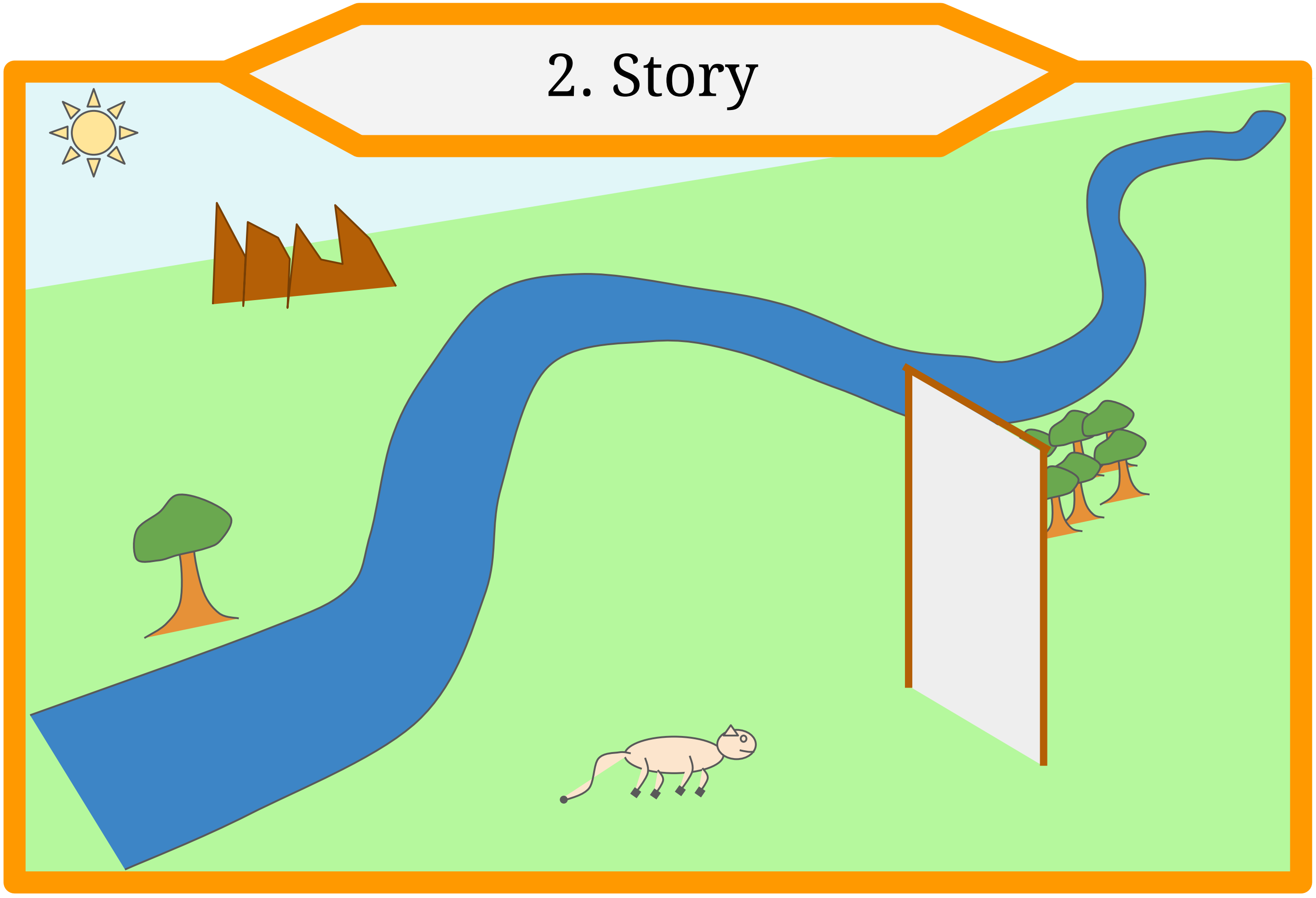
Information

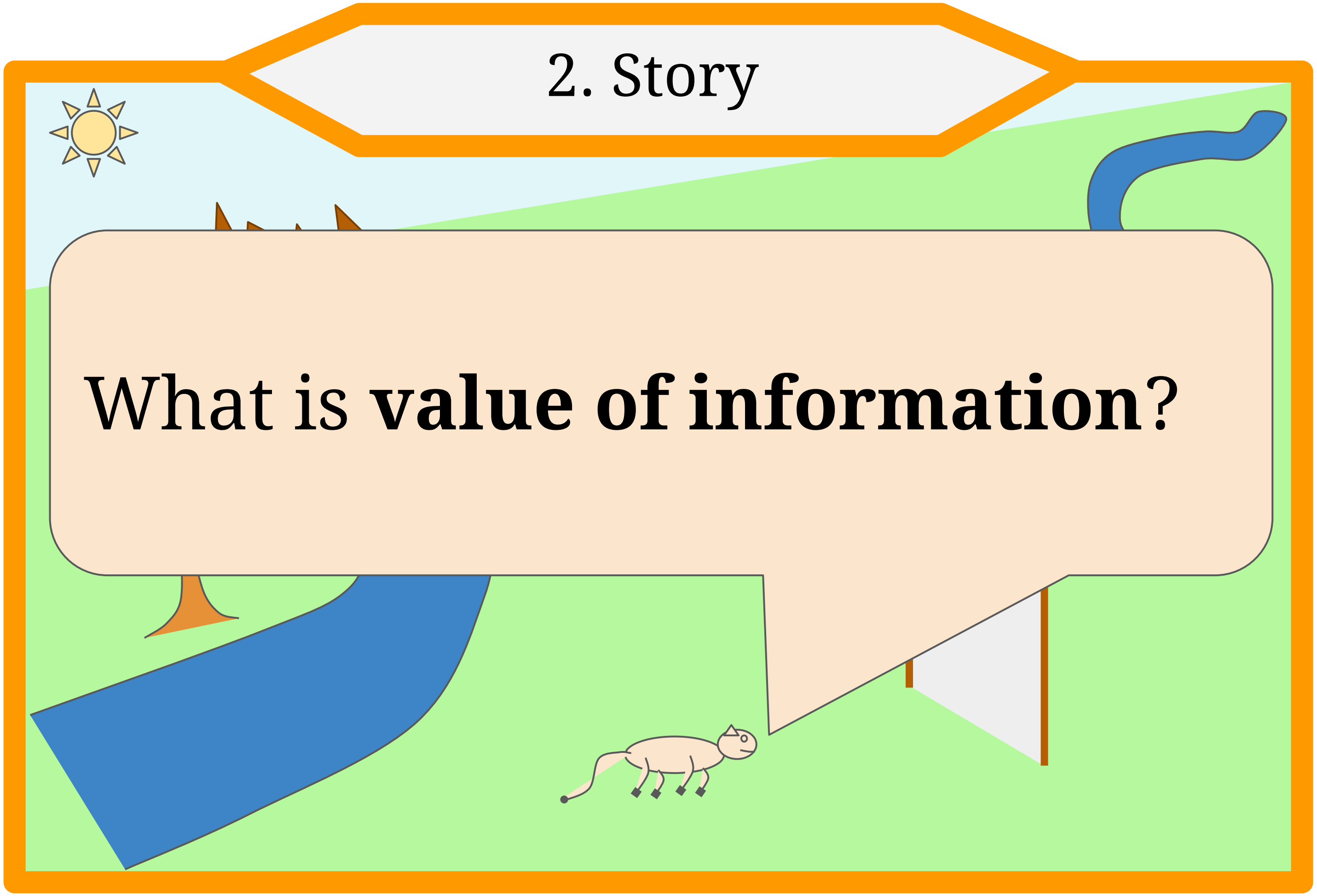
- What is the "value" of a set of pieces of information?

- Two pieces of information may be correlated, redundant, …

…

2. Story

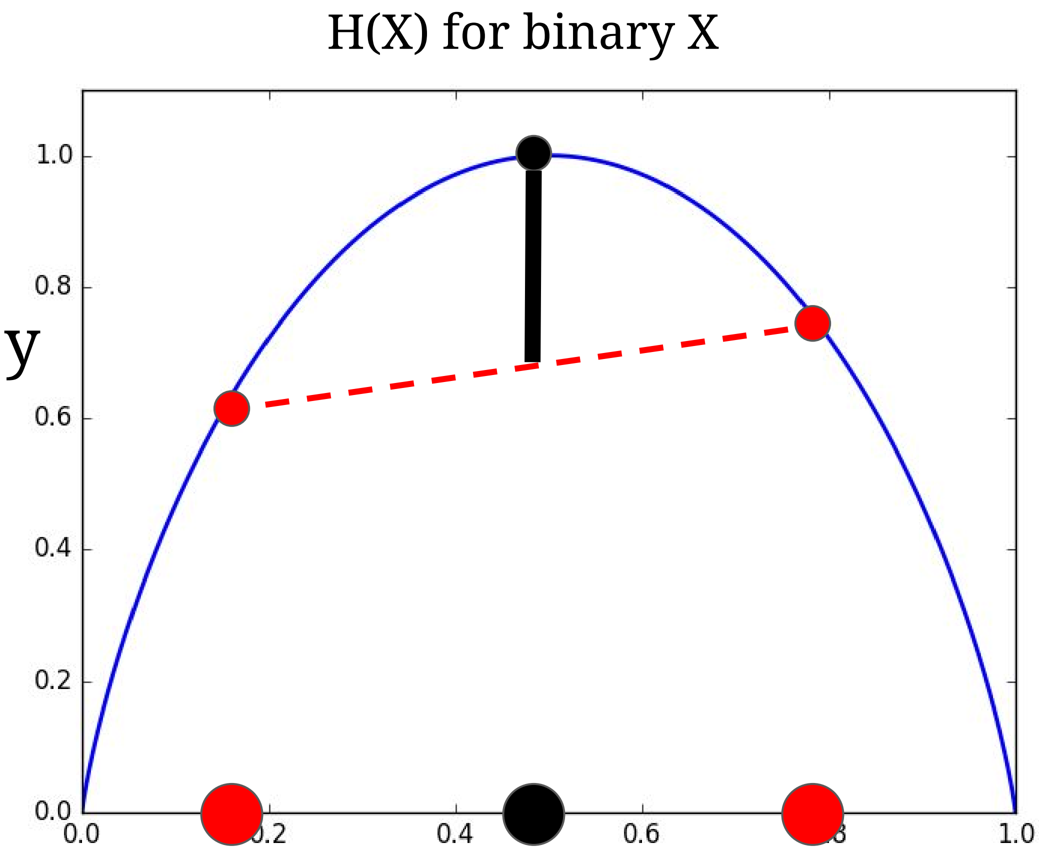



2. Story
What is value of information?

The Story part 1: Shannon 1948

- prior p on X
- posterior given Y=y

H(X) - H(X|Y)

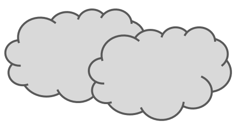


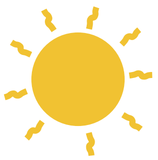
The Story part 2: Howard 1966

1. Known prior p on X

2. Select decision d

2. Nature draws x ~ p





3. Get utility u(d, x).

V(∅) = "expected utility when deciding optimally with **no signals**"

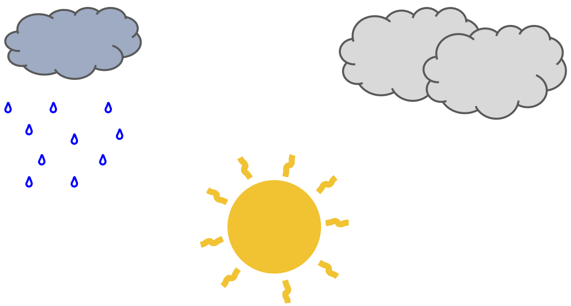
The Story part 2: Howard 1966

1. Known prior p on X

1.5. **Observe Y**, Bayesian update to p_y

2. Select decision d

2. Nature draws x ~ p_y



3. Get utility u(d, x).

V(**Y**) = “expected utility when deciding optimally after **observing Y**”

The Story part 2: Howard 1966

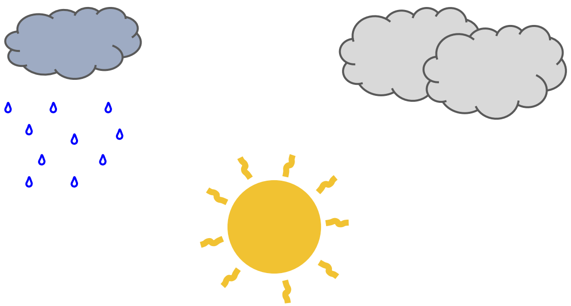
1. Known prior p on X

1.5. **Observe Y**, Bayesian update to p_y

---

2. Select decision d

2. Nature draws $x \sim p_y$



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3. Get utility u(d, x).

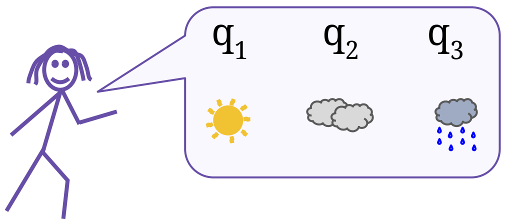
V(**Y**) = "expected utility when deciding optimally after **observing Y**"

V(Y) - V(∅) = "marginal value of Y"

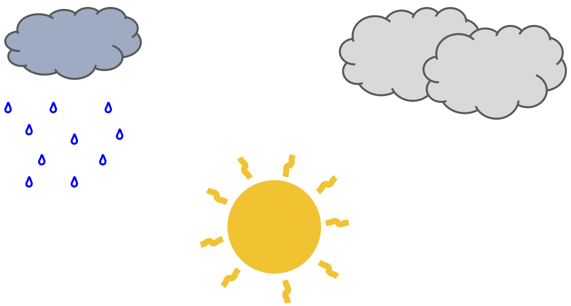
The Story part 3: Savage 1971, “scoring rules”

1. Known prior p on X

2. Select **prediction q**



2. Nature draws x ~ p



3. Get utility **S(q, x)**.

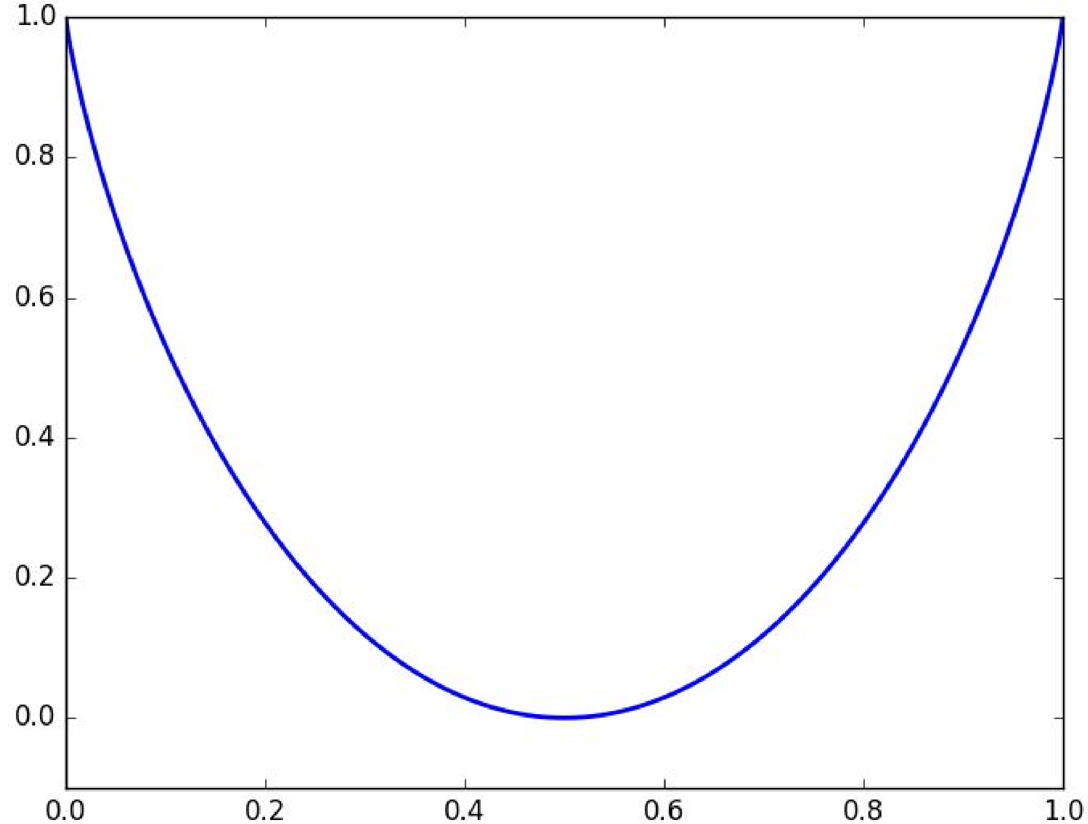
“Proper scoring rule” - optimal prediction is true belief

Example: $S(q, x) = \log q(x)$.

Savage: scoring rules $\longleftrightarrow$ convex functions

Example: $S(q, x) = \log q(x)$.

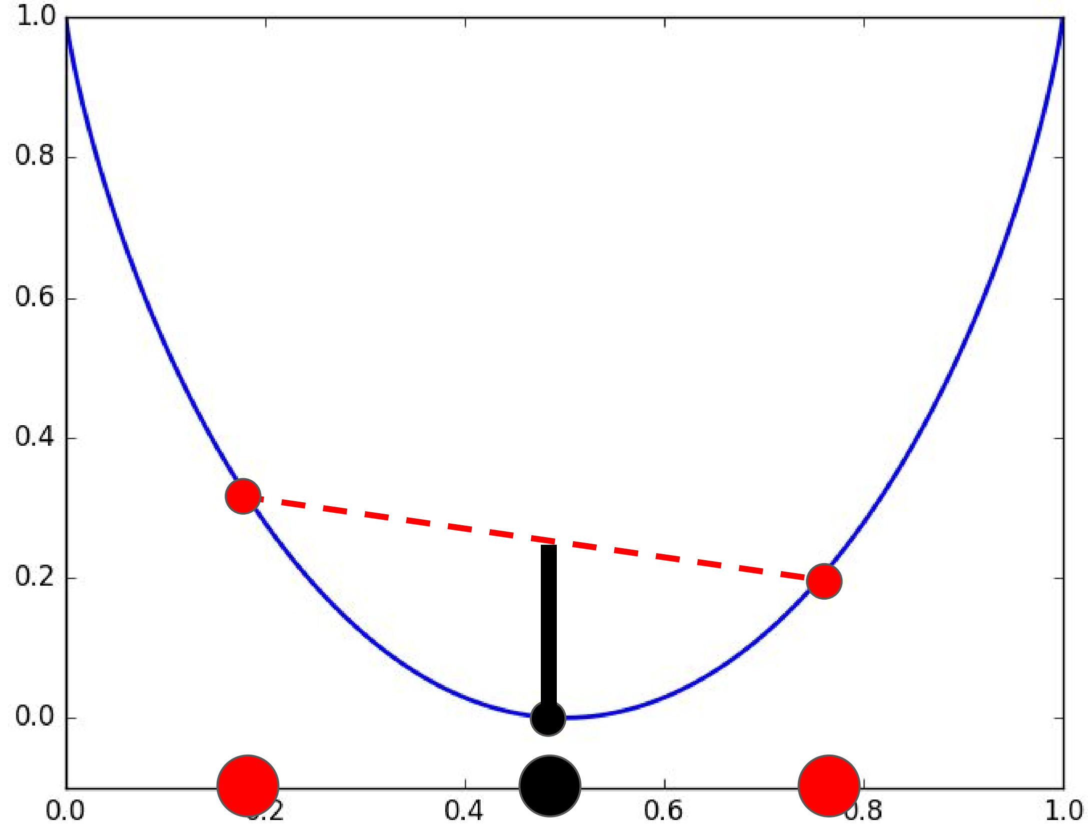
Expected score as function of belief
for binary X



Savage: scoring rules $\longleftrightarrow$ convex functions

Example: S(q, x) = log q(x).

- prior p on X
- posteriors p_y
- $V(Y) - V(\varnothing)$

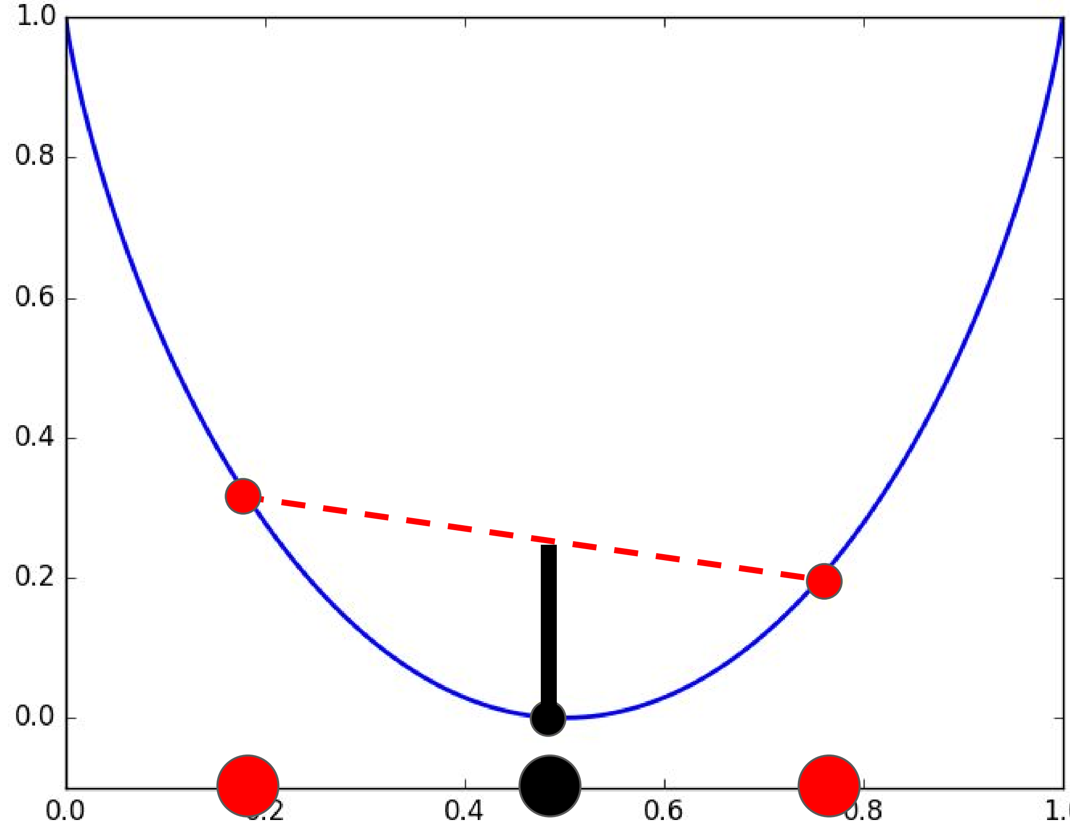


DECISION PROBLEMS $\longleftrightarrow$ convex functions!

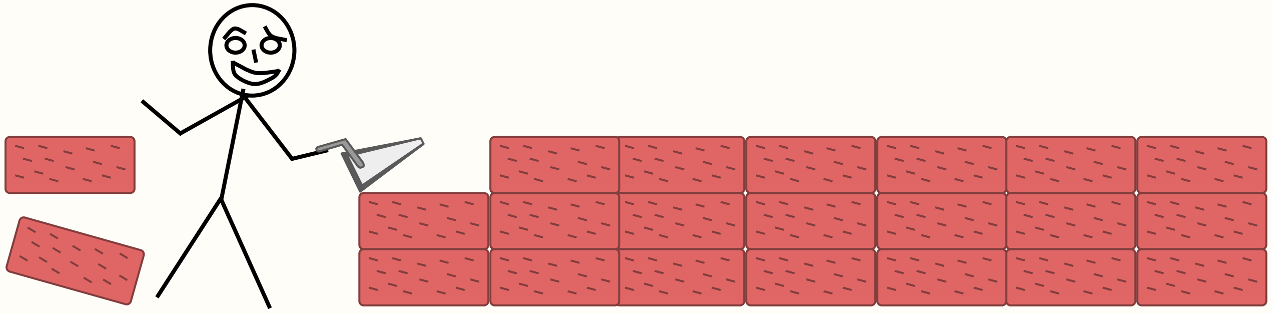
Example: S(q, x) = log q(x).

- ● prior p on X
- ● posteriors p_y
- ▌ V(Y) - V(∅)

Expected score as function of belief for binary X



3. Definitions

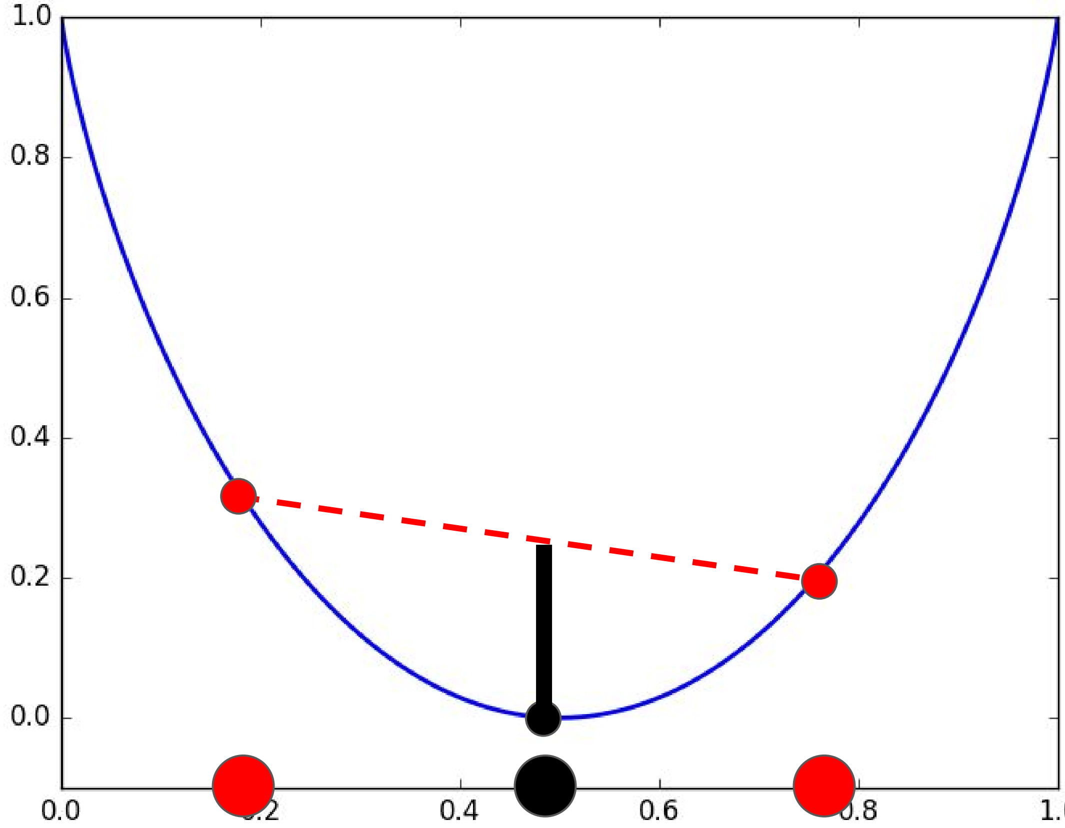


Visualizing substitutes for log scoring rule

Example: $S(q, x) = \log q(x)$.

- prior p on X
- posteriors p_y
- $V(Y) - V(\varnothing)$

Expected score for predicting X as function of posterior belief

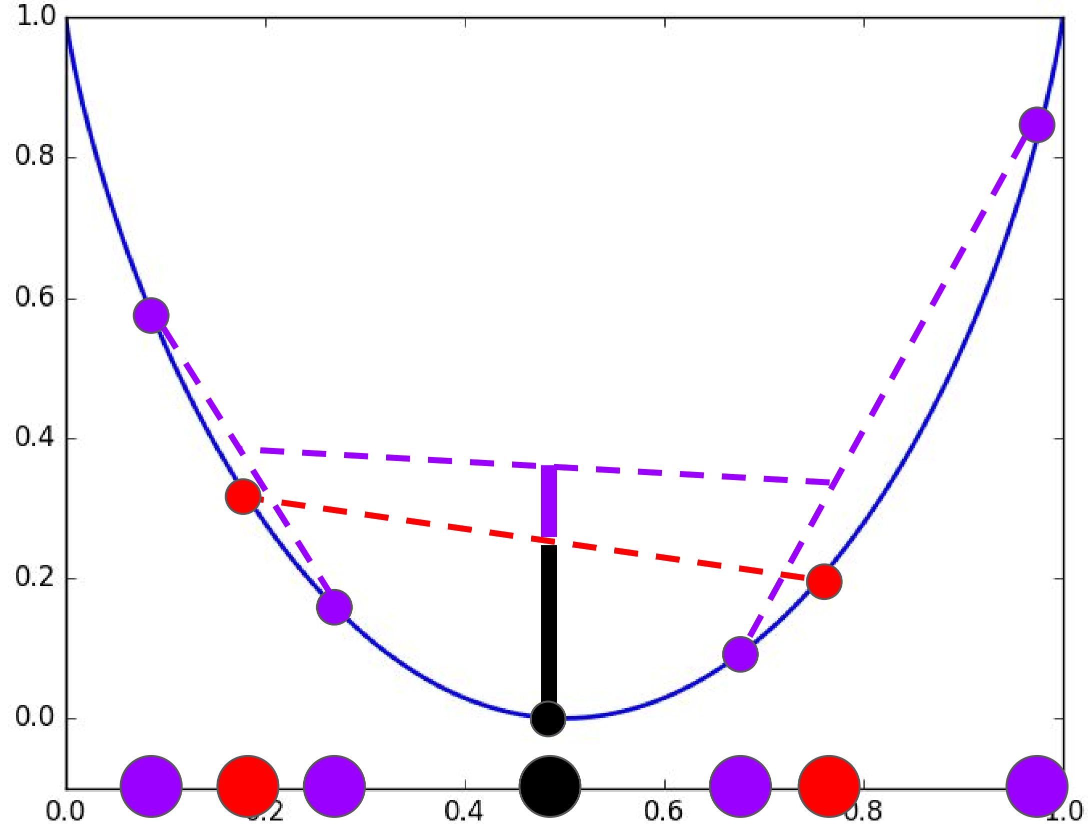


Visualizing substitutes for log scoring rule

Example: $S(q, x) = \log q(x)$.

- prior p on X
- posteriors p_y
- $V(Y) - V(\varnothing)$
- posteriors p_{yz}
- $V(Y,Z) - V(Y)$

Expected score for predicting X
as function of posterior belief



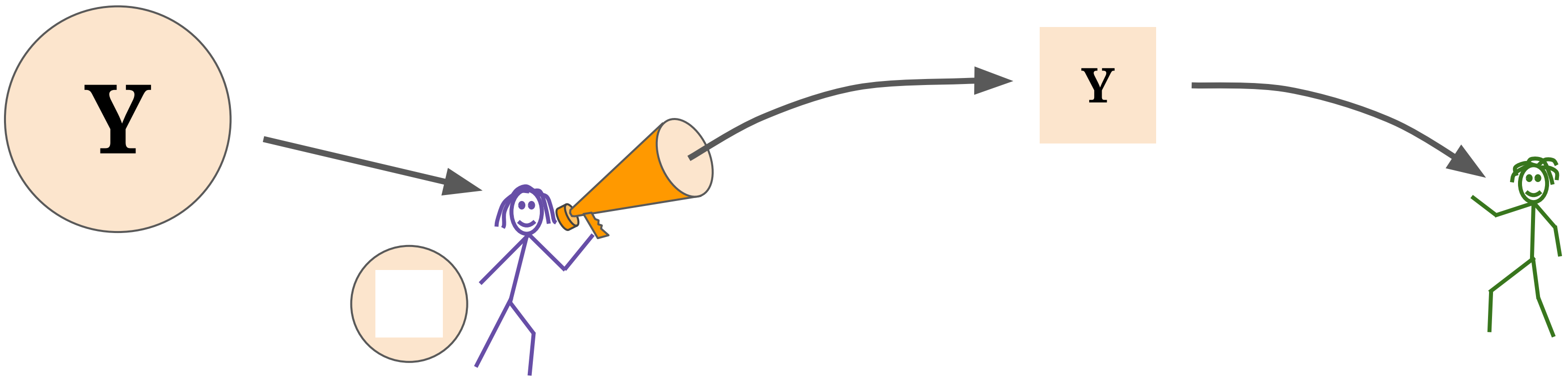
Our definitions

$Y_1 \ldots Y_n$ are **substitutes** for u if V is **submodular**:
For $A \subseteq B \subseteq \{Y_1 \ldots Y_n\}$,

$$V\big(A \cup \{Y_i\}\big) - V(A) \geq V\big(B \cup \{Y_i\}\big) - V(B).$$

- complements = supermodular
- depends on **both decision prob AND info structure**

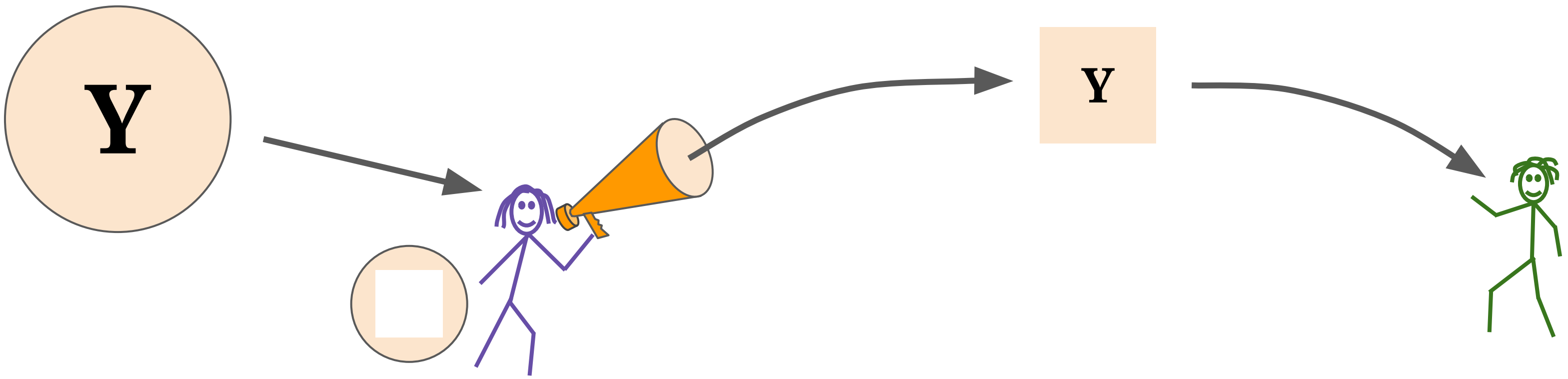
Roadblock: Information is divisible!



"Half the truth is often a great lie."

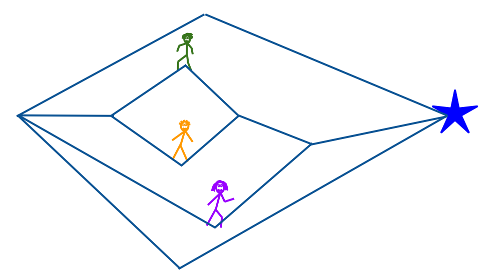
- Benjamin Franklin

Roadblock: Information is divisible!



“Half the truth is often a great lie.”
- Benjamin Franklin

Solution: extend definitions.
(See my TCS+ talk on **Nov. 9**!)



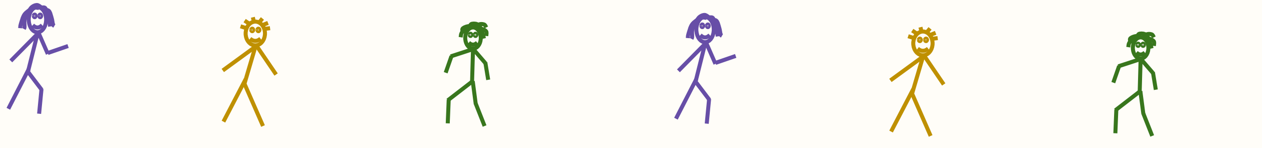
-- Remainders --

Two separate applications:

- Markets (for information).
  substitutes $\longleftrightarrow$ good equilibria

- Algorithms.
  complexity of optimal info. acquisition

4. Prediction markets



Idea / motivation

Each agent has a signal Y_i.

Goal: aggregate into prediction about X **quickly**.

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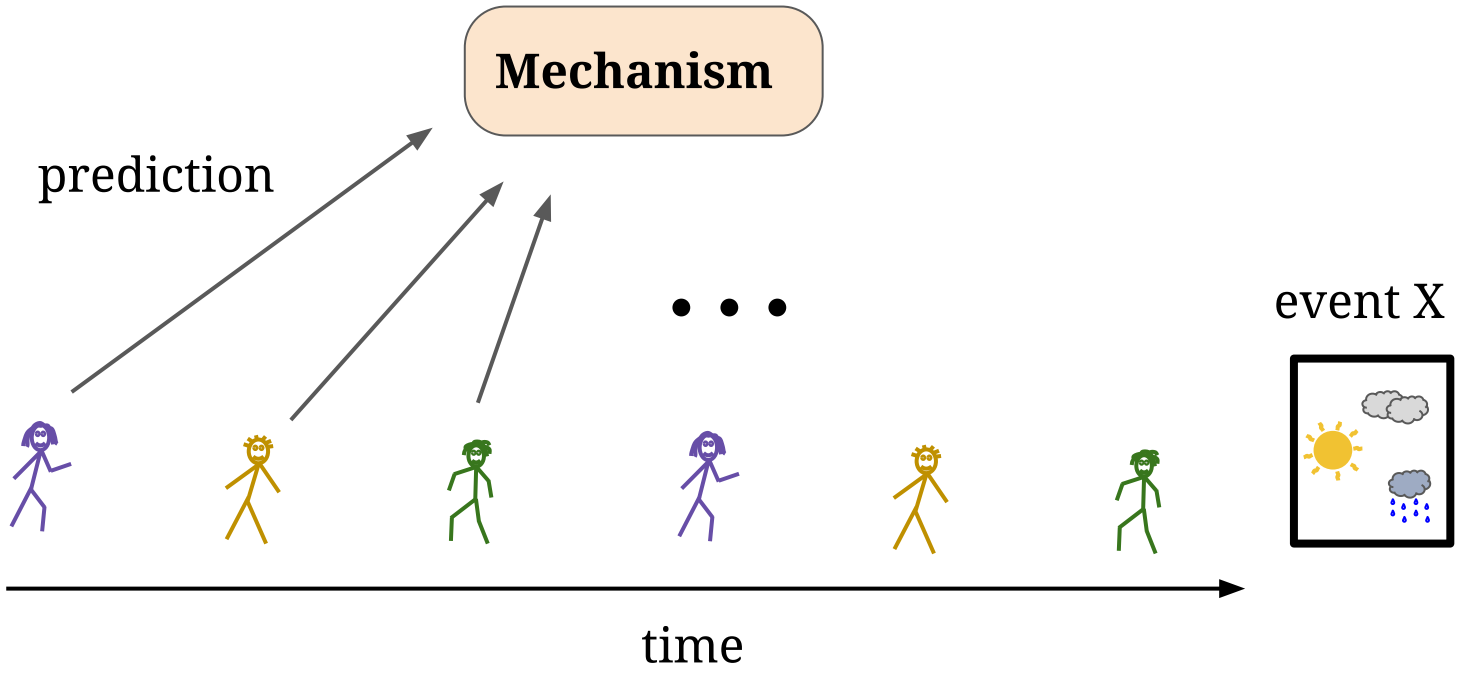
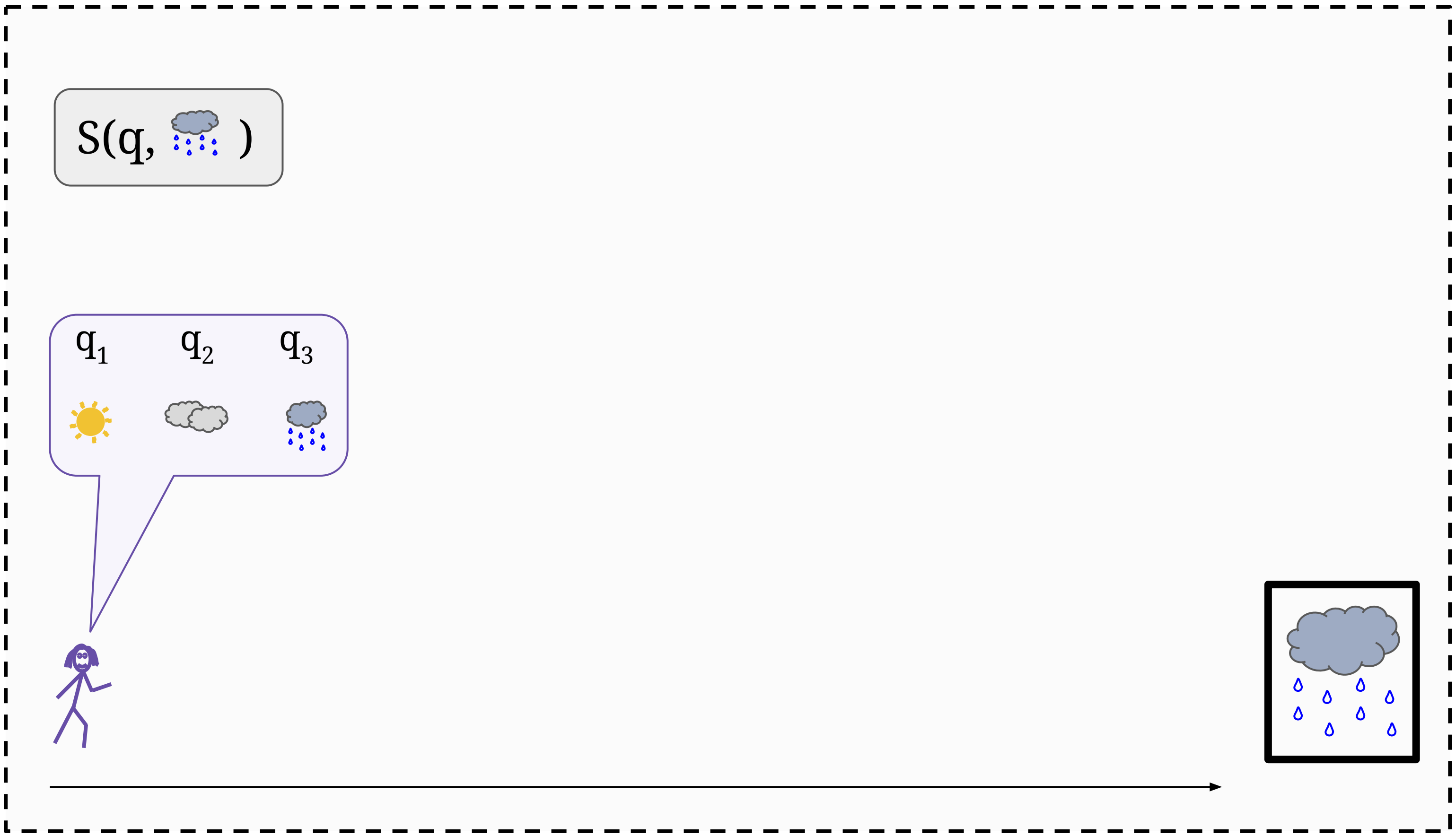


The mechanism [Hanson 2003]*

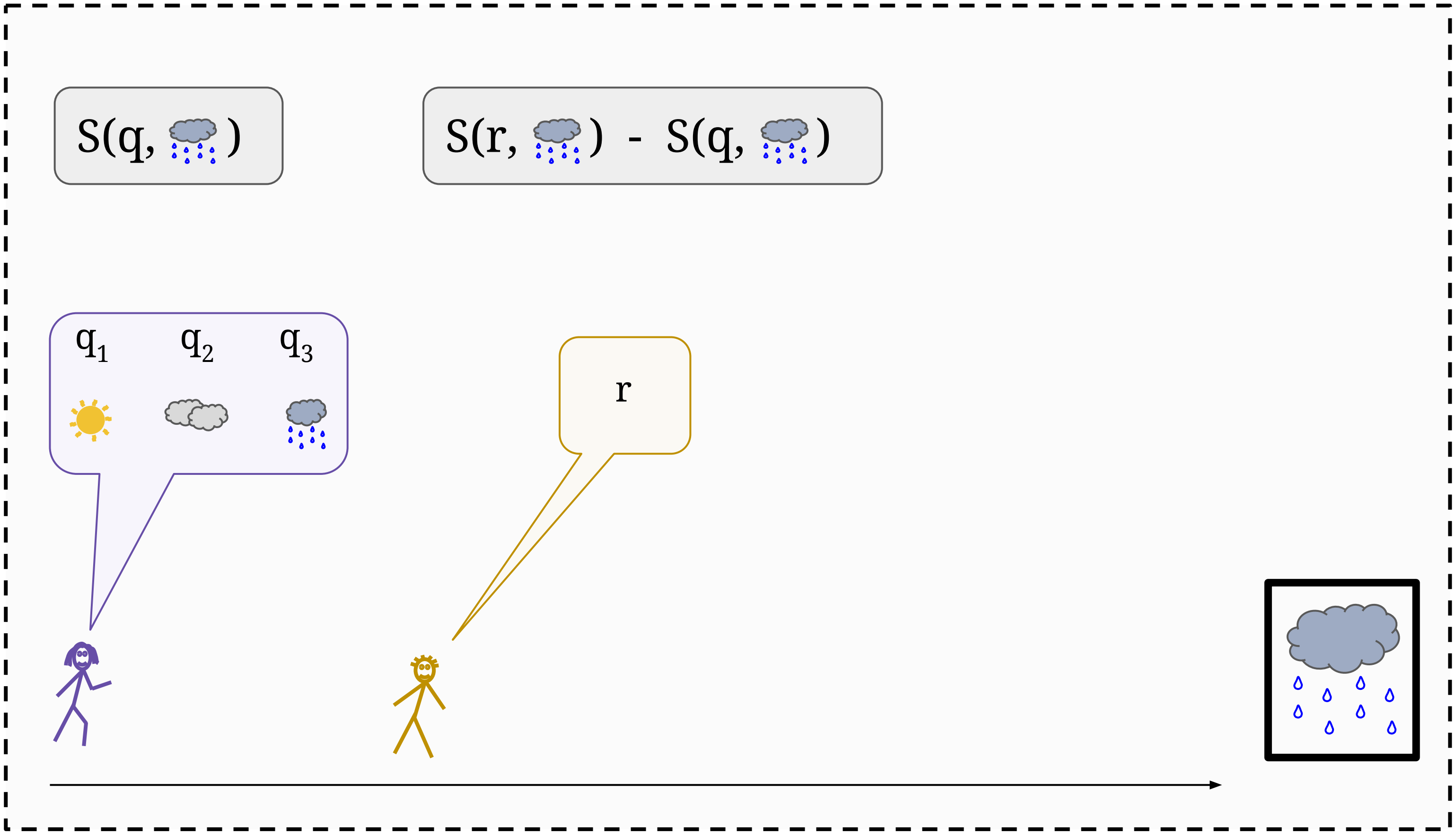
Only one participant: proper scoring rule! Truthful.



*can also be viewed as buying/selling shares [Abernethy+Chen+Wortmann-Vaughan 2013]

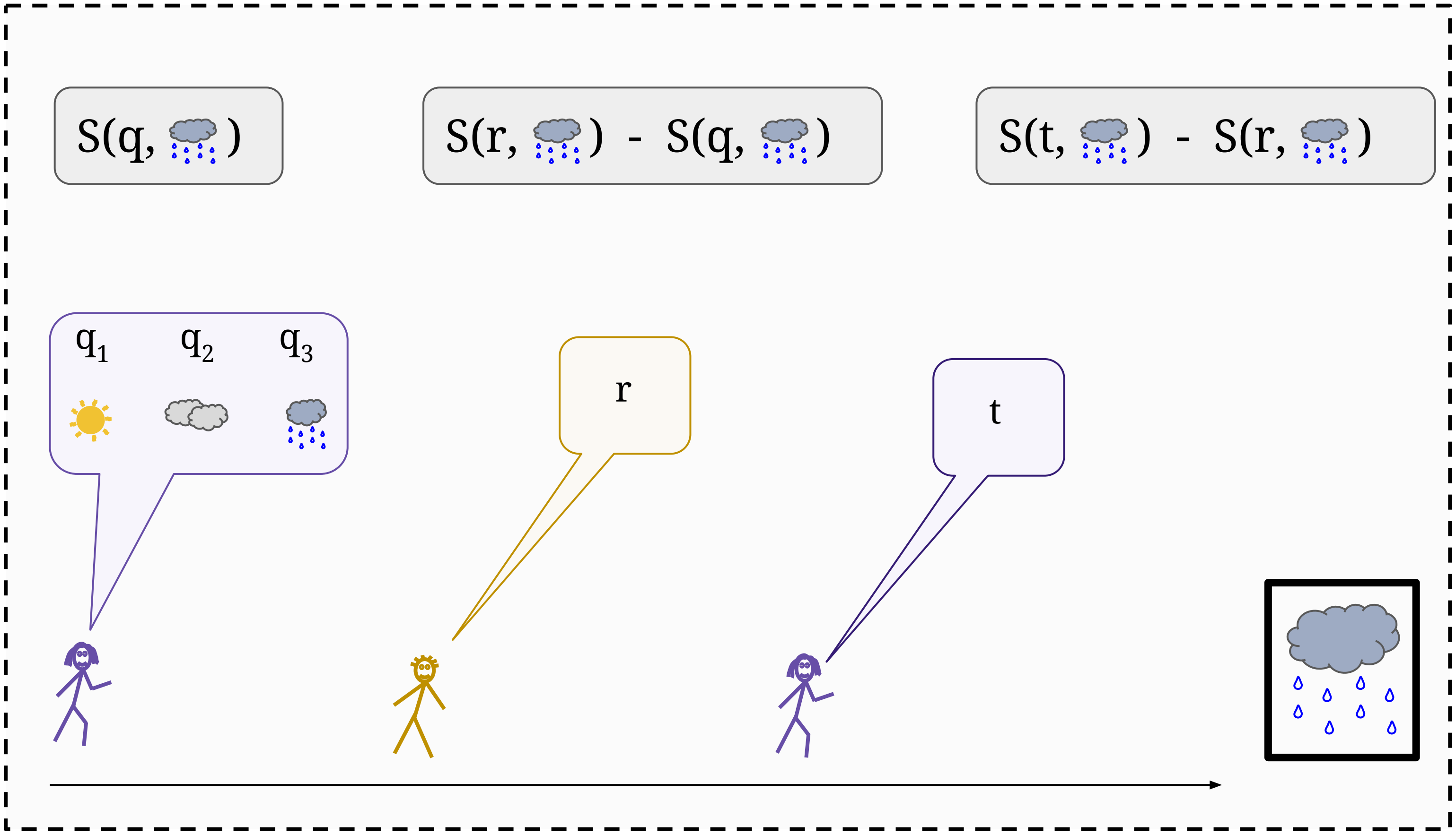
The mechanism [Hanson 2003]

Two participants: “chained” scoring rule! Truthful.



The mechanism [Hanson 2003]

Two participants, three stages: **not understood!**



The mechanism [Hanson 2003]

Two participants, three stages: **not understood!**

Known: for **log scoring rule**, if $Y_1 \ldots Y_n$ are…

- conditionally independent on X ⇒ “rush”.
  [Chen+Dimitrov+Sami+Reeves+Pennock+Hanson+Fortnow+Gonen 2010]
- independent ⇒ “delay”.
  [Gao+Zhang+Chen 2013]

Prediction markets results

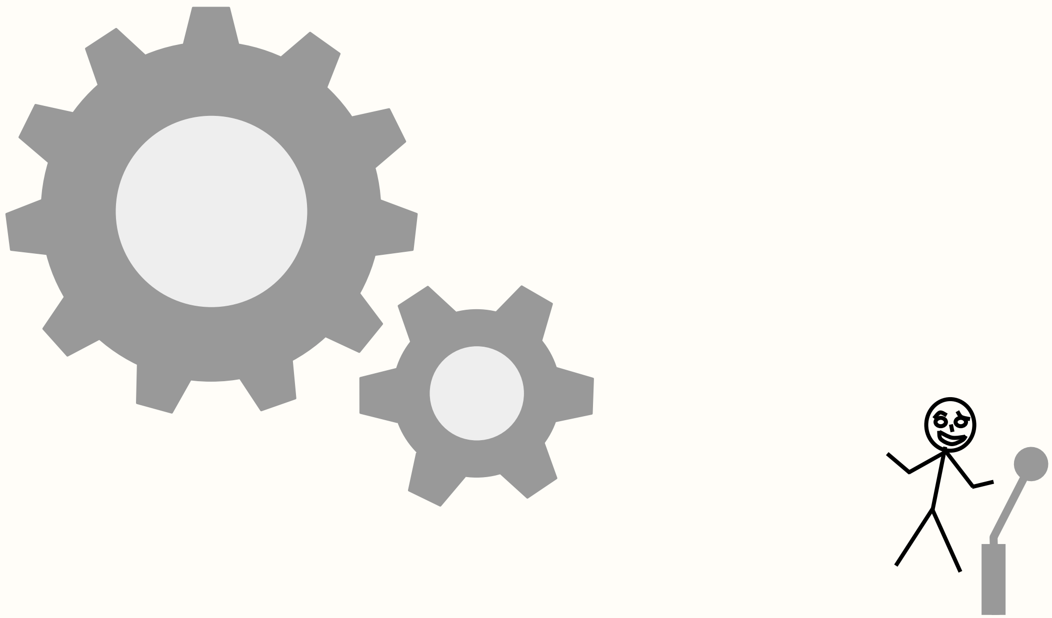
Thm. If and only if signals are strong **substitutes**, the only equilibria are "**all rush**".

(efficient market hypothesis $\longleftrightarrow$ substitutes)

Thm. If and only if signals are strong **complements**, the only equilibria are "**all delay**".

(market failure $\longleftrightarrow$ complements)

5. Algorithms



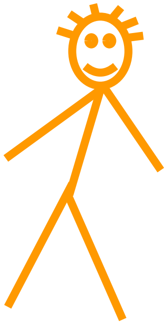
Algorithmic question “SignalSelection”

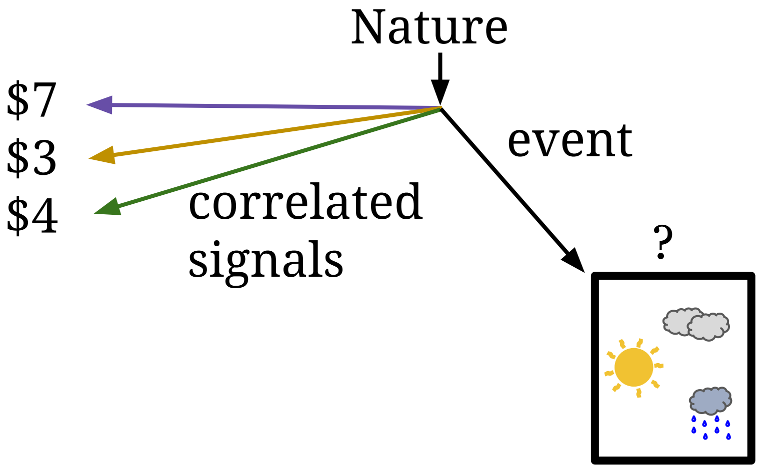
Input:

- utility function u (as an oracle…)
- joint distribution $X, Y_1 \ldots Y_n$ (as an oracle…)
- prices $\pi_1 \ldots \pi_n$ for the signals, budget constraint B

Output:

- which signals to acquire





Complexity results

Reduction: SignalSelection → set function maximization.

Substitutes ⇒ 1-1/e approx in polynomial time

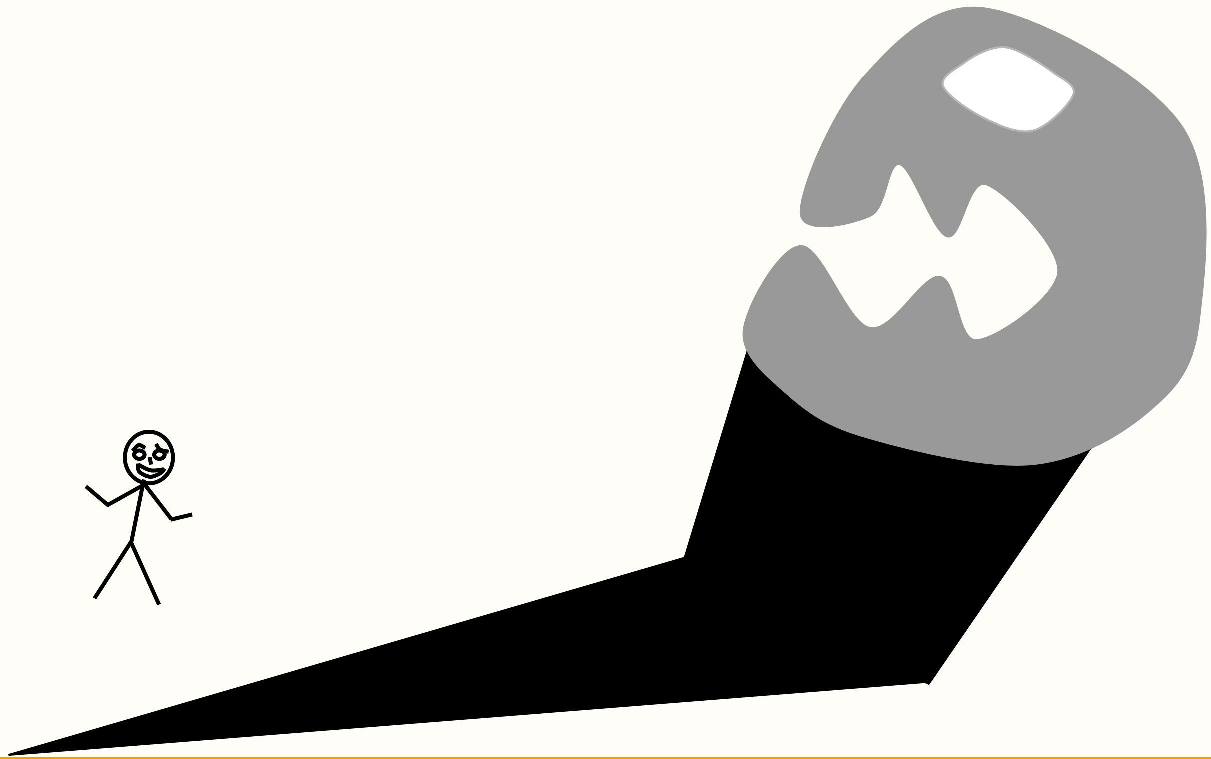
Reduction: set function maximization → SignalSelection.

Comps/generally ⇒ no approx w/ subexp. queries

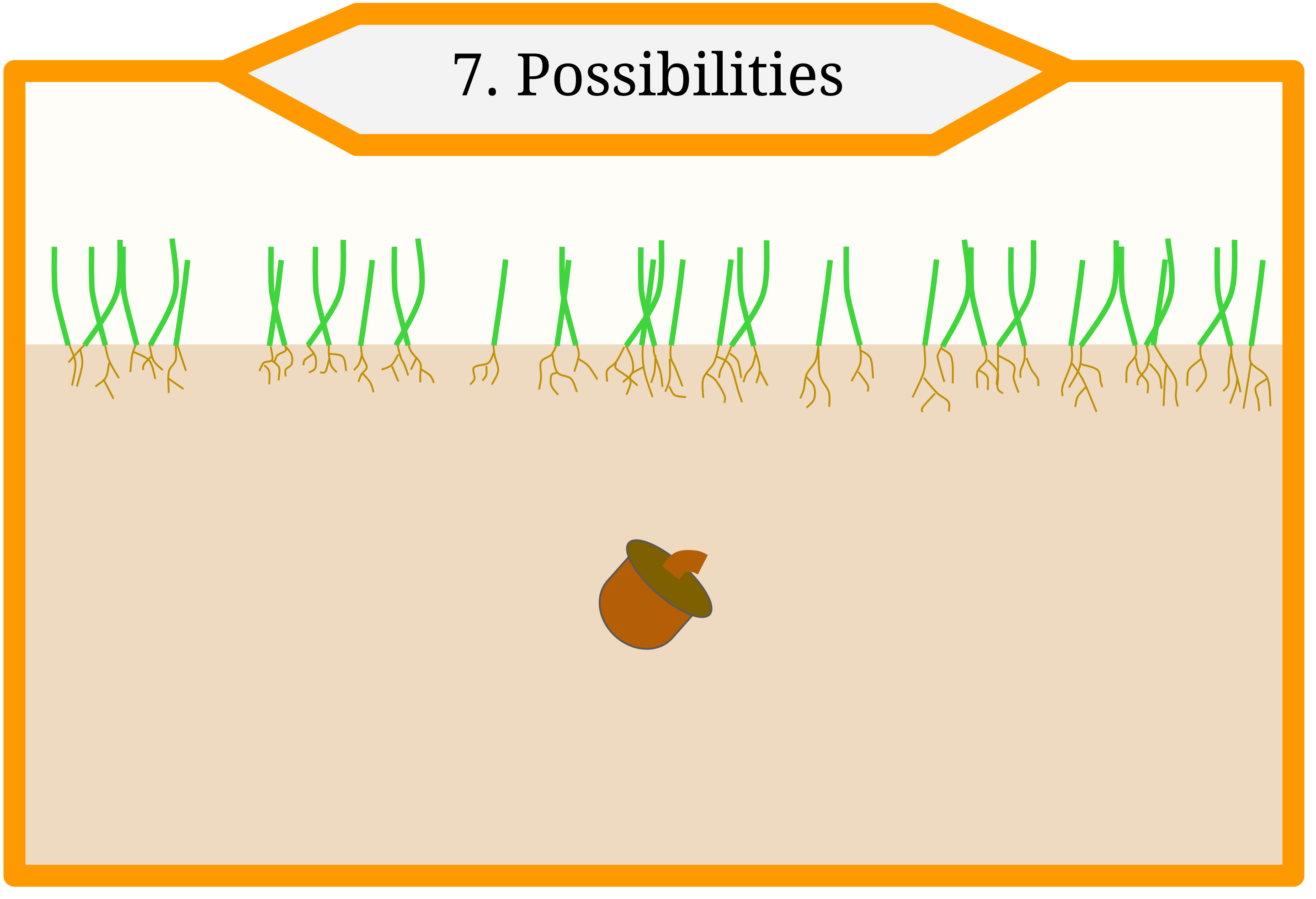
Notes:

- As in submod. maximization, can handle e.g. matroid constraints.
- Ideas not new here at all! See [Krause+Guestrin 2011]
- Model / generality, focus of our question are new

6. Shadows



7. Possibilities



Open problems (small selection)

Game theory

- selling information
- signalling
- bundling complements
- other useful applications?

Algorithms

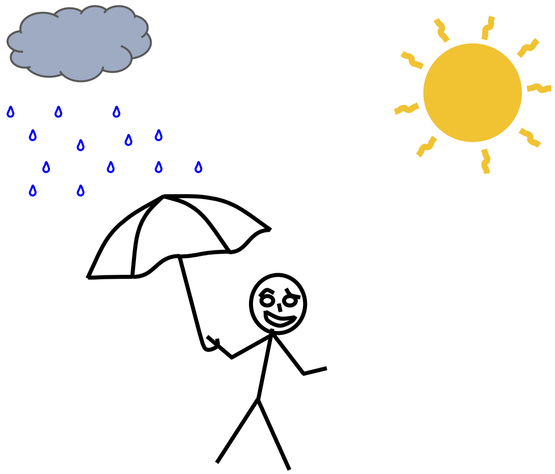
- check if signals are strong substitutes
- compute Alice's best response in stage one (decompose signal into sub. and comp. components)
- SignalSelection on discrete or continuous lattices!

Structure

- examples of (classes of) subs and comps
- "more substitutable" signal structures? utilities?
- "universal" substitutes and complements
- connections: e.g. sensitivity of Boolean functions?

Resources

TCS+, my talk on **Nov. 9**	sites.google.com/site/plustcs/
These slides:	bowaggoner.com/
Blog posts on proper scoring rules, generalized entropies, …	bowaggoner.com/blog/
Information elicitation slides:	sites.google.com/site/informationelicitation/



Thanks!

extra slides

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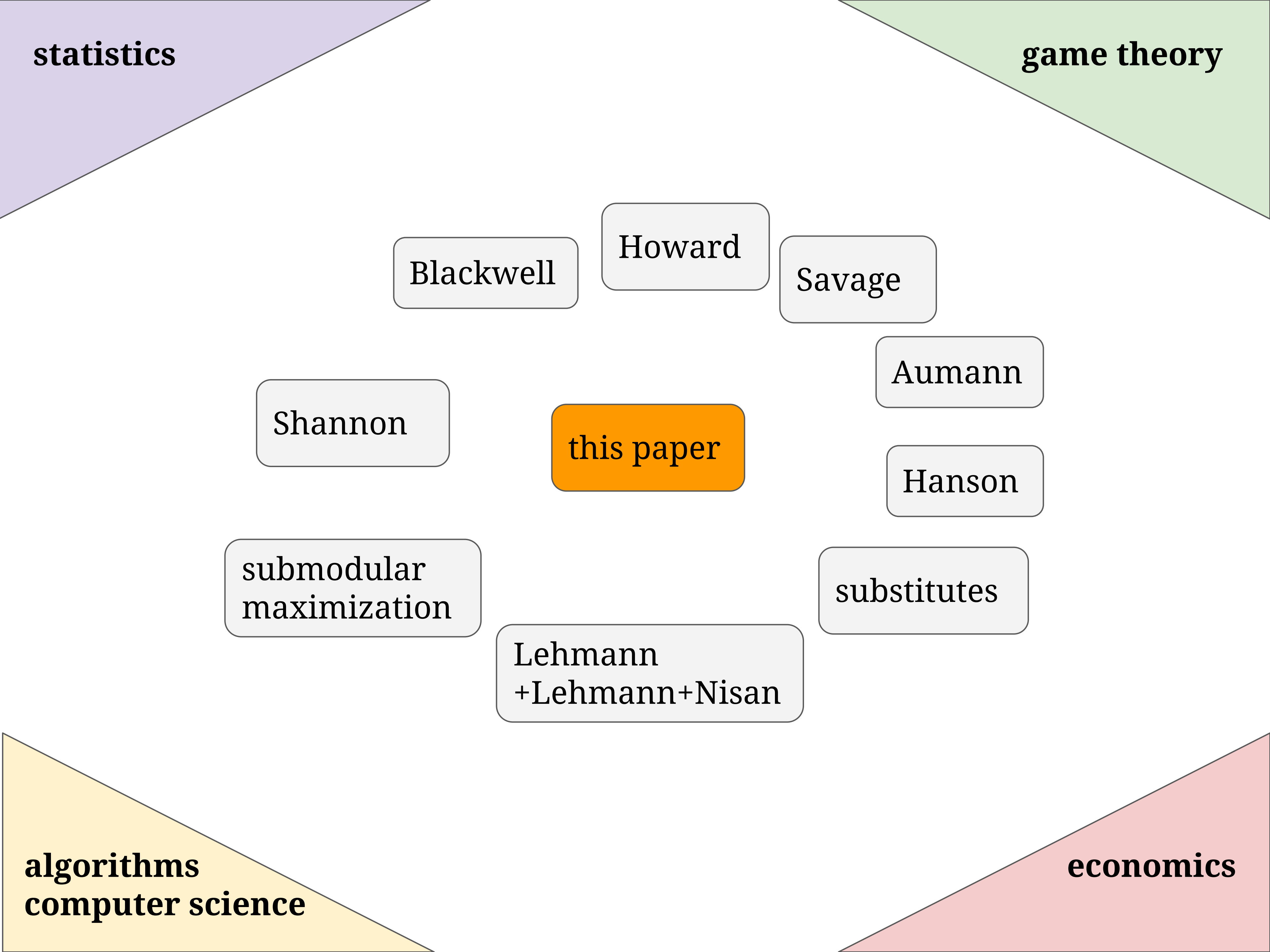
The 57th Annual Symposium on Foundations of Computer Science (FOCS 2016), sponsored by the IEEE Computer Society Technical Committee on Mathematical Foundations of Computing, will be held in New Brunswick, New Jersey on October 9-11 (Sunday through Tuesday).
On Saturday, October 8th, FOCS will join the celebration of Avi Wigderson's 60th birthday.

Papers presenting new and original research on theory of computation are sought. Typical but not exclusive topics of interest include: algorithms and data structures, computational complexity, cryptography, computational learning theory, economics and computation, parallel and distributed algorithms, quantum computing, computational geometry, computational applications of logic, algorithmic graph theory and combinatorics, optimization, randomness in computing, approximation algorithms, algorithmic coding theory, algebraic computation, and theoretical aspects of areas such as networks, privacy, information retrieval, computational biology, and databases. Papers that broaden the reach of the theory of computing, or raise important problems that can benefit from theoretical investigation and analysis, are encouraged.


statistics
game theory
Blackwell
Howard
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this paper
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submodular maximization
substitutes
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