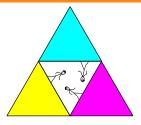
# Information Elicitation and Design of Surrogate Losses



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Peking University October 30, 2020

Based on joint work with Jessie Finocchiaro and Rafael Frongillo (U. Colorado, Boulder).

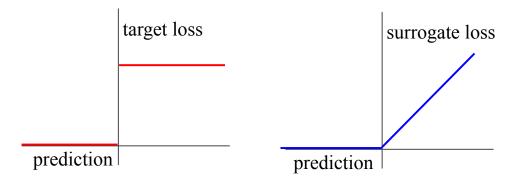
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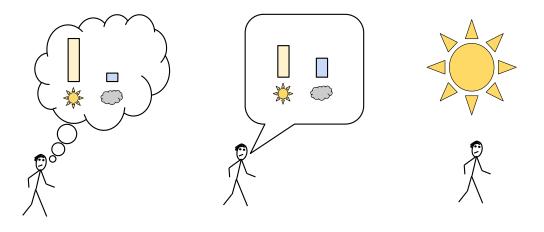
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#### **Connection between problems**

 $\mathop{\mathrm{argmin}}_{r\in\mathcal{R}} \mathbb{E}_{Y\sim p} \ell(r,Y)$ 

# Outline

- **1** Concepts and definitions from information elicitation what do you get when you minimize a loss?
- 2 Surrogate loss functions for machine learning
- 3 The embedding approach; our contributions

Part 1: Concepts and definitions from information elicitation

What do you get when you minimize a loss?

$$\Gamma(p) := \underset{r \in \mathcal{R}}{\operatorname{argmin}} \underset{Y \sim p}{\mathbb{E}} \ell(r, Y)$$
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Examples:

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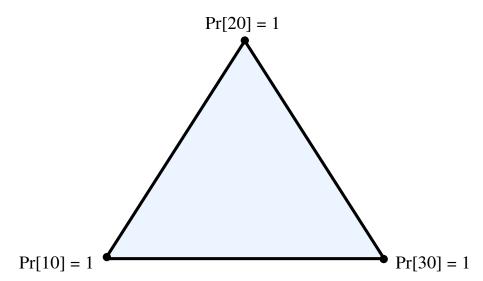
$$\Gamma(p) := \underset{r \in \mathcal{R}}{\operatorname{argmin}} \underset{Y \sim p}{\mathbb{E}} \ell(r, Y)$$
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•  $\Gamma: \Delta_{\mathcal{Y}} \to 2^{\mathcal{R}}$  is a **property** of the distribution p.

•  $\Gamma$  is elicitable if there exists  $\ell$  such that (1) holds.

#### Information elicitation - the picture

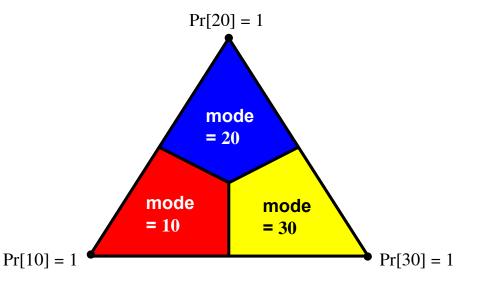
The simplex  $\Delta_{\mathcal{Y}}$  for  $\mathcal{Y} = \{10, 20, 30\}$ :



#### Information elicitation - the picture

A property is a partition of the simplex.

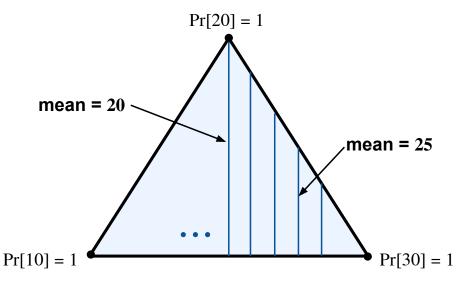
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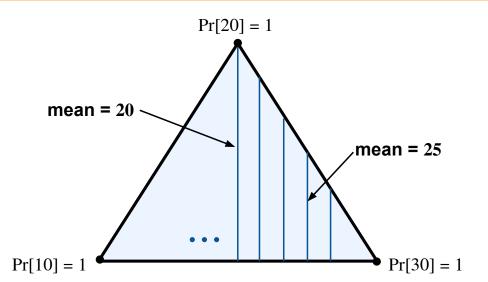
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## Key basic fact

#### Theorem

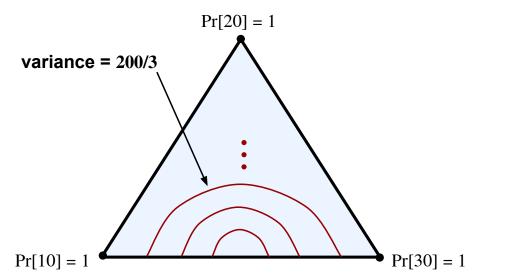
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**Elicitation complexity**<sup>1</sup> of  $\Gamma$ : fewest parameters needed to indirectly elicit  $\Gamma$ . *Variance: 2.* 

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Part 2: Surrogate loss functions for machine learning

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Let  $p_x$  = conditional distribution of Y given X = x. Bayes optimal:  $h(x) = \gamma(p_x)$ where  $\gamma$  is the property elicited by  $\ell$ .

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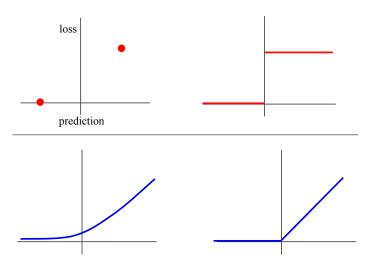
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- Our point (new work): it is necessary and almost sufficient for  $L, \psi$  to indirectly elicit  $\gamma$ . Lower bounds, etc.

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Part 3: The embedding approach; our contributions.

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*Proof idea:* Use the convex conjugate of the Bayes risk of  $\ell$ . *Not really a new construction; e.g. prediction markets!* 

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More generally: works!

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Amazing embedding construction:<sup>3</sup>  $d = \lceil \log_2 |\mathcal{Y}| \rceil$ .

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In fact, there exists  $\epsilon > 0$  and C > 0 such that, for all u and p,

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**Implication:** Fast rates of convergence of L translate *linearly* to fast rates for  $\ell$ . Not generally true for smooth surrogates. Summary and supplementary results

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Other results on tractability:

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