Contracts with Information Acquisition, via Scoring Rules

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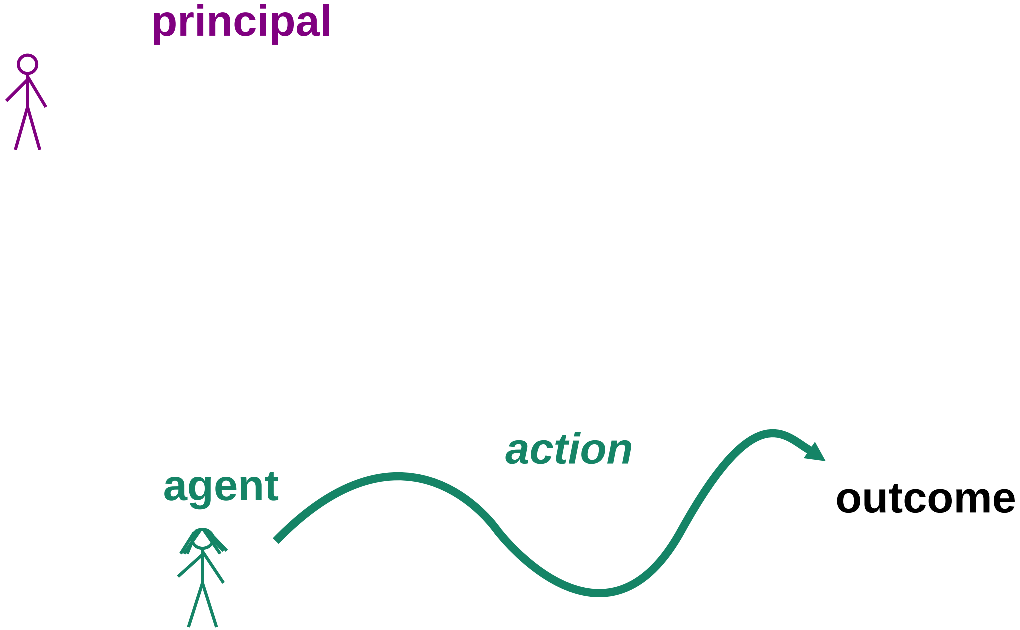
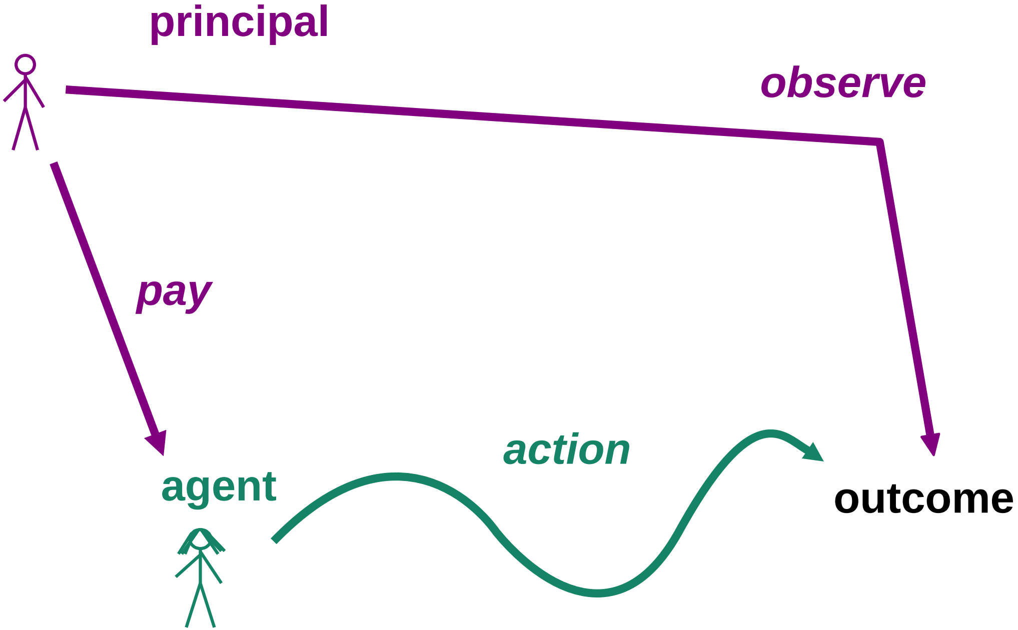
University of Colorado, Boulder

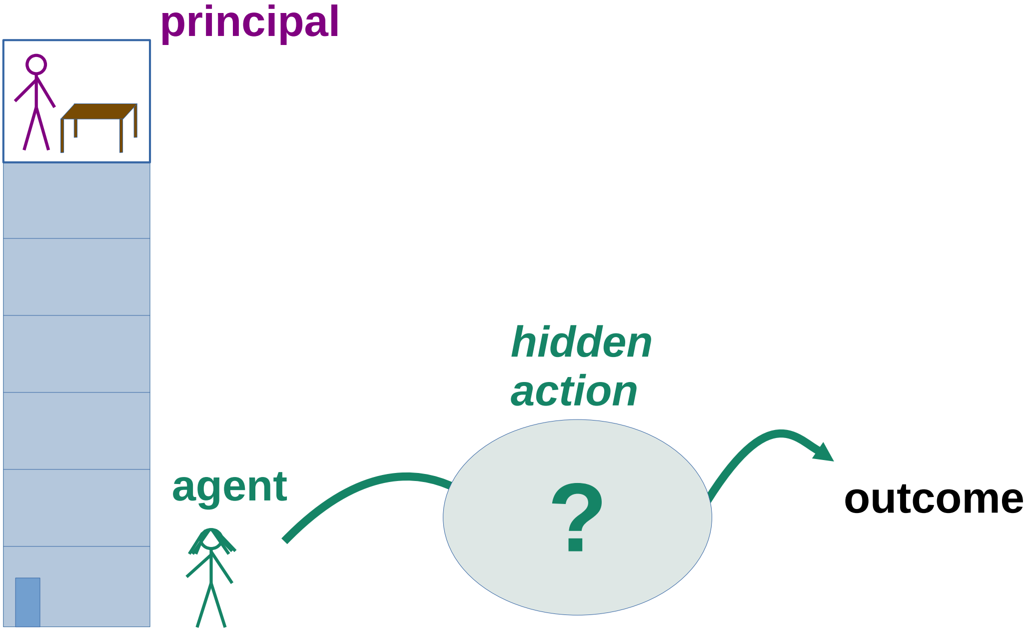
TheoryFest
June 2022

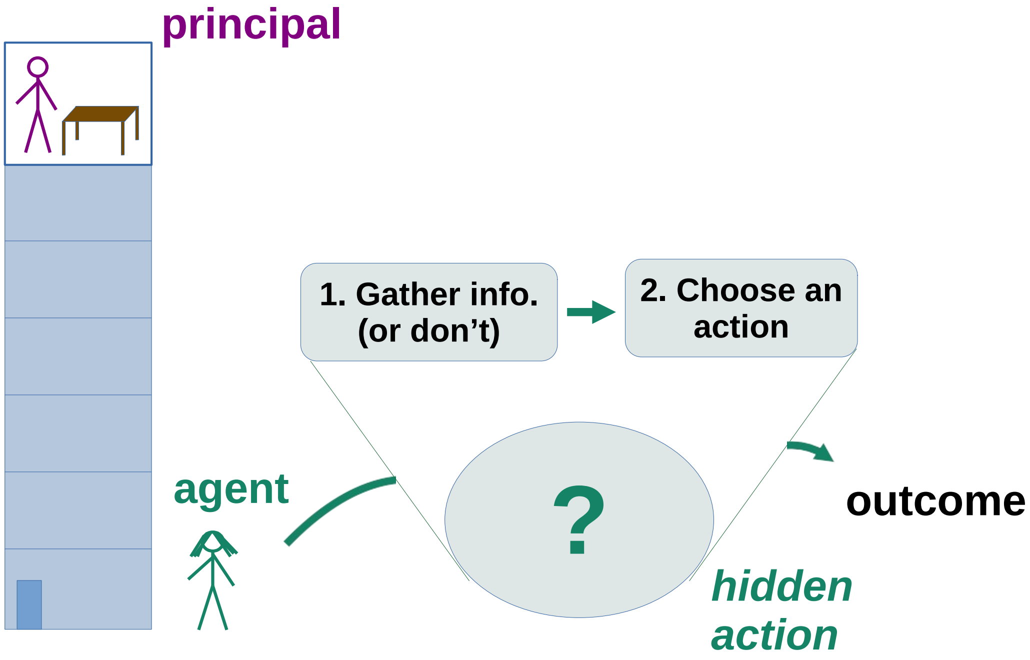
principal

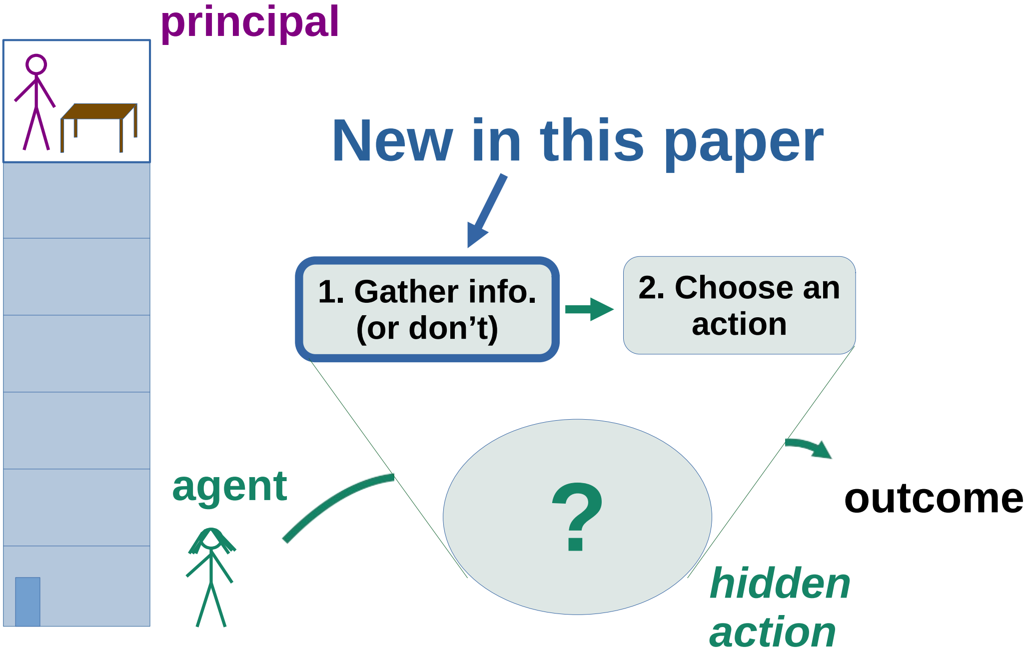
agent

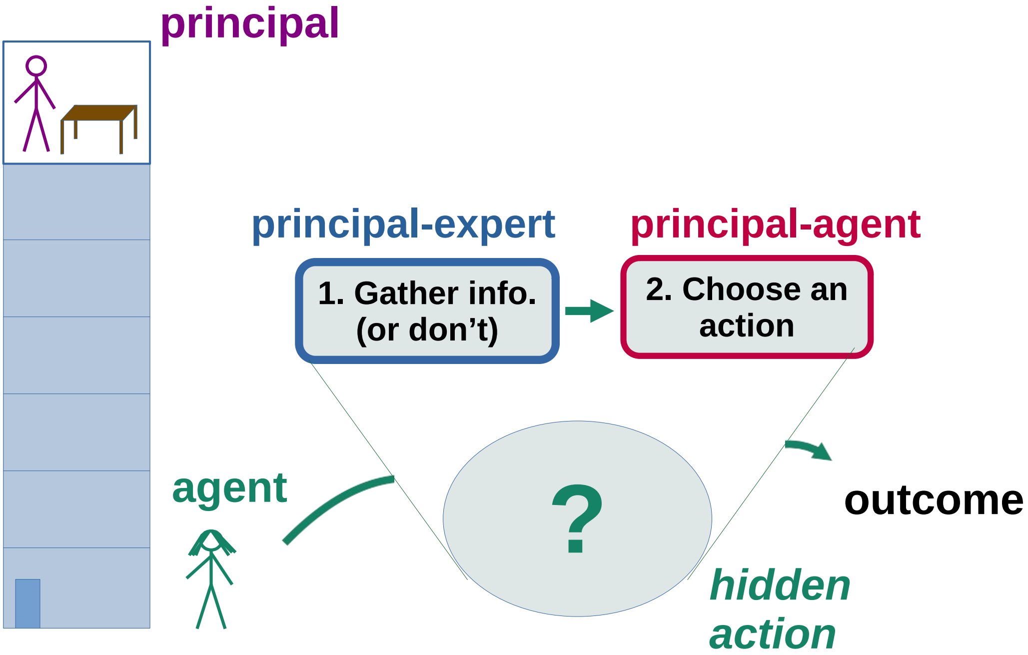

principal
agent
action
outcome


principal
observe
pay
action
agent
outcome


principal
hidden
action
?
agent
outcome


principal
1. Gather info.
(or don't)
2. Choose an
action
agent
?
outcome
hidden
action


principal
New in this paper
1. Gather info.
(or don't)
2. Choose an
action
agent
?
outcome
hidden
action



Zermeño (2012); Boutilier (2012); Oesterheld and Conitzer (2020)

Outline

1. Model
2. Reducing to design of scoring rules
3. Special cases
4. Main result

1. Model

Plan

1. Revisit standard **hidden action** model
2. Recall **information acquisition** model
3. Give **our model**

Hidden Action - Model

1. Principal offers a **contract** $t : \Omega \to \mathbb{R}$
2. Agent chooses an action a, incurs cost c_a
3. Outcome $\omega \sim p_a$ revealed, payment $t(\omega)$.

Hidden Action - Model

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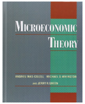
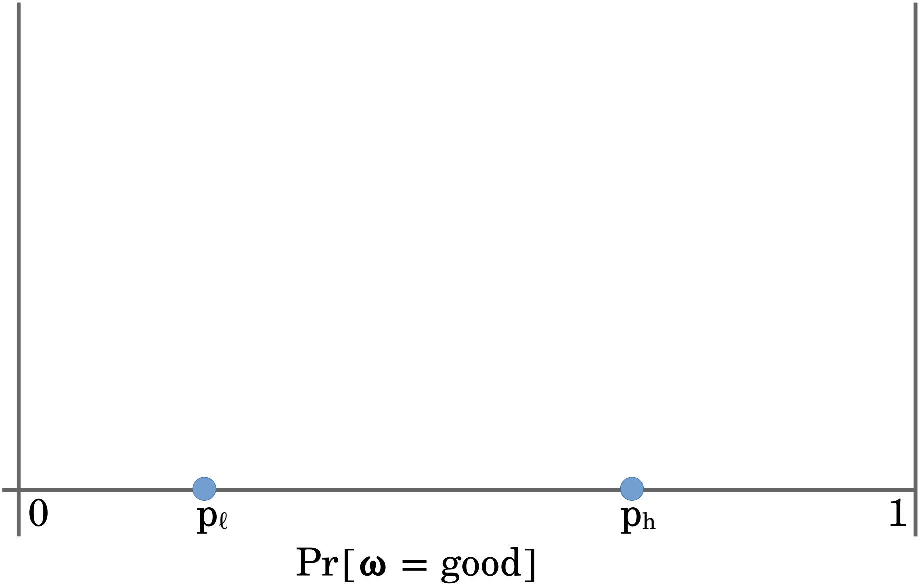
Require **limited liability:** payment ≥ 0 always.

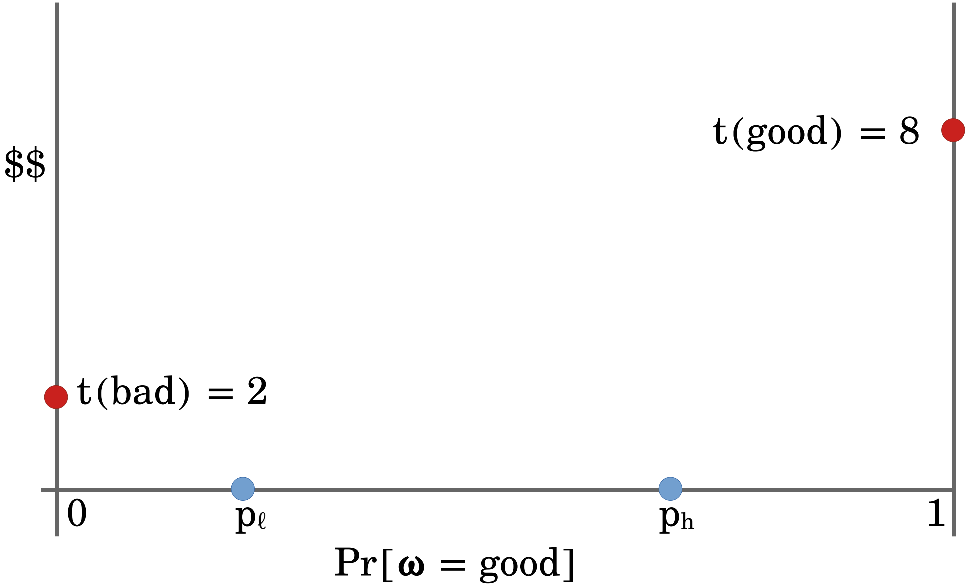
Minimimum payment problem: incentivize a as cheaply as possible

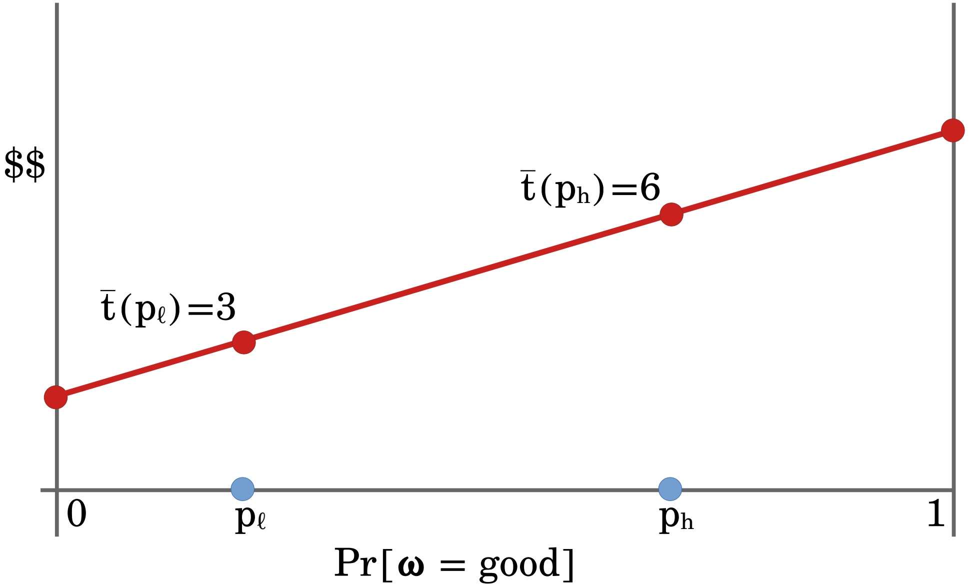
agent maxes $\mathbb{E}$ utility

Related work: 

, `https://sites.google.com/view/ec22-act-workshop`


0
p_ℓ
p_h
1
$\Pr[\boldsymbol{\omega} = \text{good}]$


$$
t(good) = 8
t(bad) = 2
0
p_ℓ
p_h
1
Pr[ω = good]


$$
$\bar{t}(p_h)=6$
$\bar{t}(p_\ell)=3$
0
p_ℓ
p_h
1
Pr[ω = good]

Information acquisition - model[1]

[1] Related work: Li et al. 2022; Chen and Yu 2021; discussed later.

Information acquisition - model[1]

1. Principal offers a scoring rule $s : \Delta_\Omega \times \Omega \to \mathbb{R}$
2. Agent chooses whether to acquire signal for cost κ
3. Agent reports a prediction p
4. Outcome ω is revealed, pay $s(p, \omega)$.

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Observe: prediction p yields **contract** $t(\omega) = s(p, \omega)$.

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Information acquisition - model[1] (take 2)

1. Principal offers a scoring rule $s : \Delta_\Omega \times \Omega \to \mathbb{R}$
   Principal offers a menu T of contracts
2. Agent chooses whether to acquire signal for cost κ
3. Agent reports a prediction p
   Agent selects a contract $t \in T$
4. Outcome ω is revealed, pay $s(p, \omega)$.
   Outcome ω is revealed, pay $t(\omega)$

Observe: prediction p yields **contract** $t(\omega) = s(p, \omega)$.

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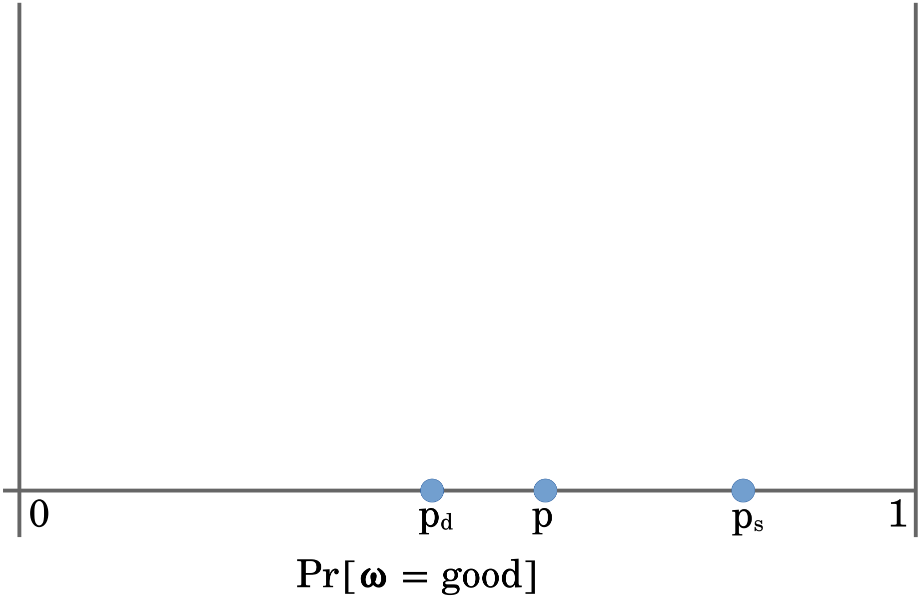
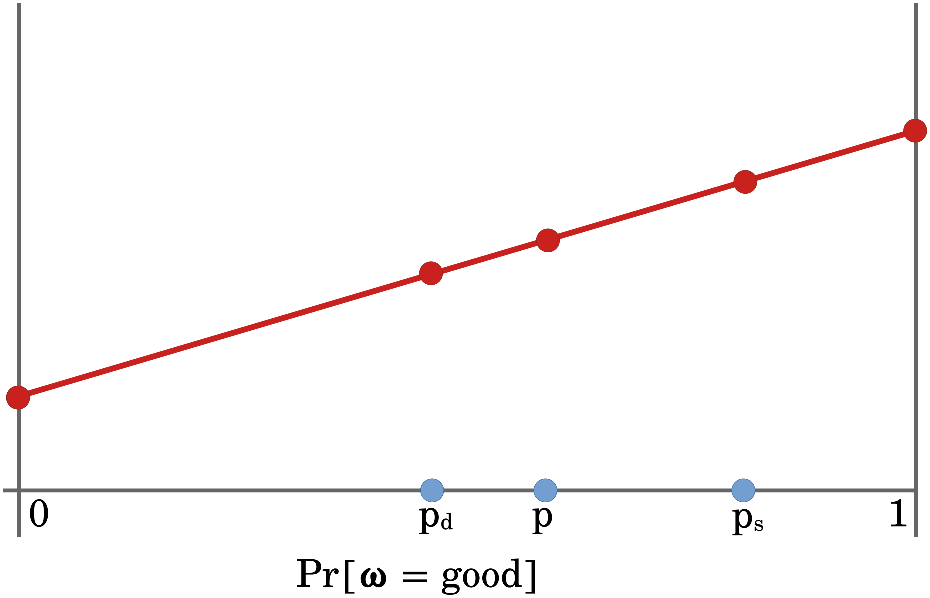
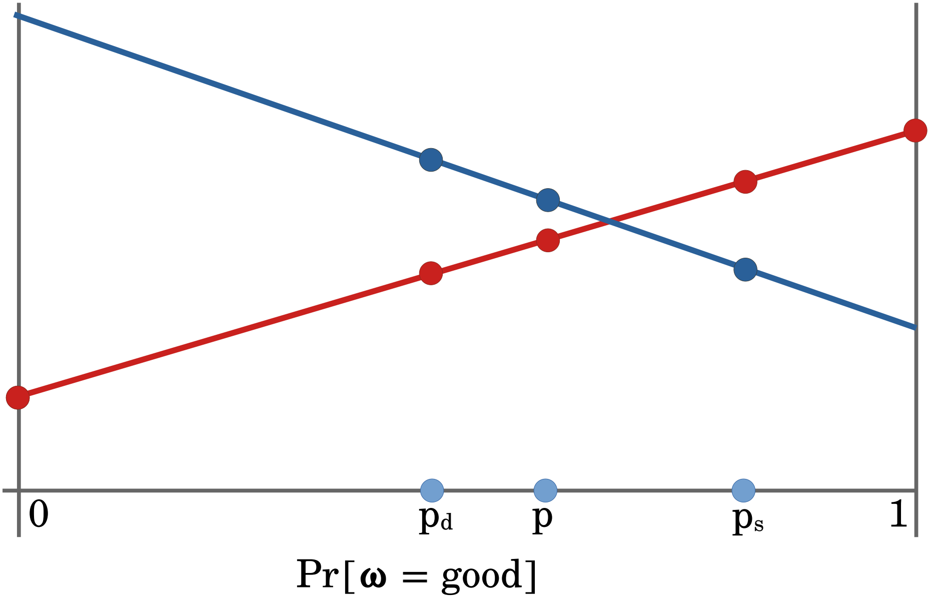
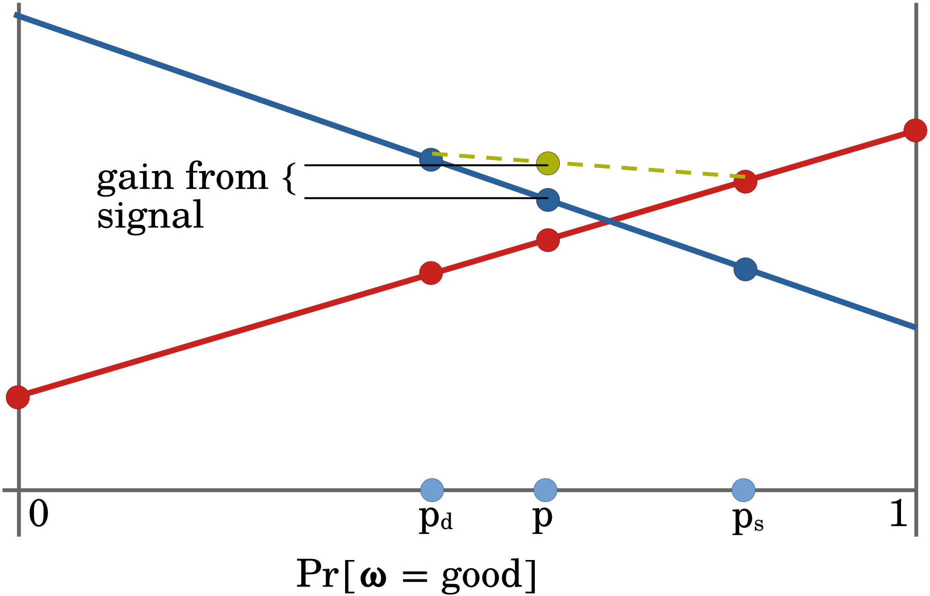

0
p_d
p
p_s
1
Pr[ω = good]


0
p_d
p
p_s
1
Pr[ω = good]


0
p_d
p
p_s
1
$\Pr[\omega = \text{good}]$


gain from
signal
0
p_d
p
p_s
1
$\Pr[\omega = \text{good}]$

Our model

Contracts with Information Acquisition:

1. Principal offers a menu T of contracts
2. Agent chooses whether to acquire signal S *cost κ*
3. Agent selects a contract $t \in T$
4. Agent selects an action a *cost c_a*
5. Outcome $\omega \sim p_{a,S}$ is revealed, pay $t(\omega)$

Our model

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Minimum payment problem:

given a **plan**, design T so the agent follows it

Our model

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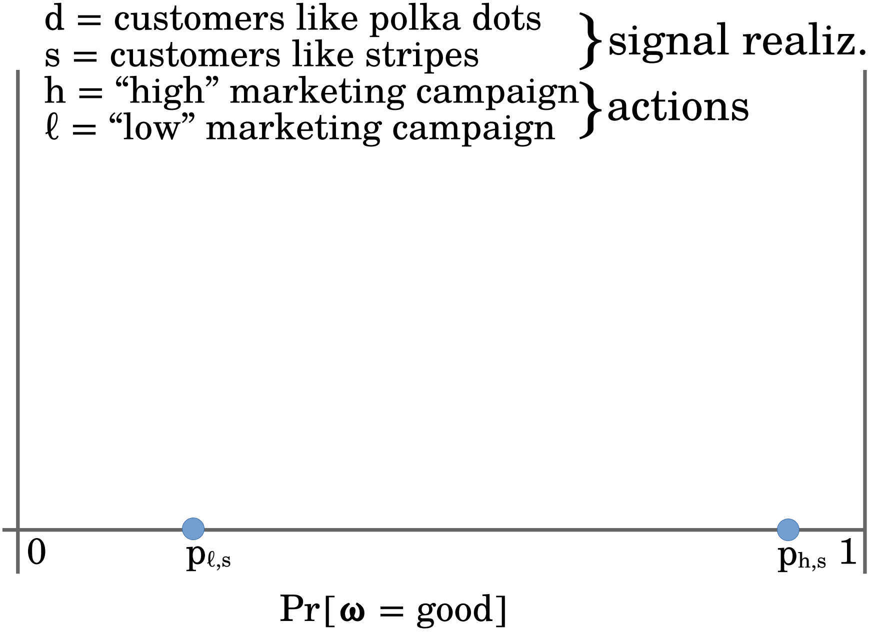
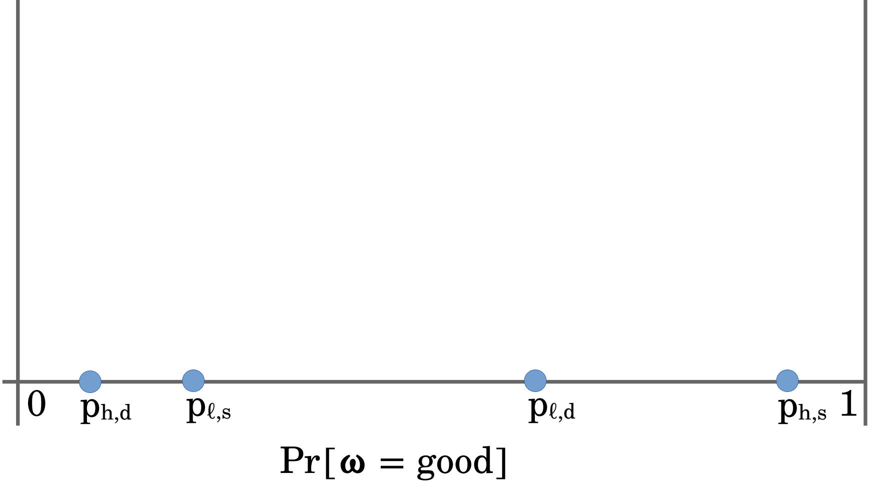
Minimum payment problem:

given a **plan**, design T so the agent follows it
i.e. minimize expected payment subject to limited liability, IC, IR.


d = customers like polka dots
s = customers like stripes
signal realiz.
h = "high" marketing campaign
ℓ = "low" marketing campaign
actions
0
$p_{\ell,s}$
$p_{h,s}$
1
Pr[ω = good]

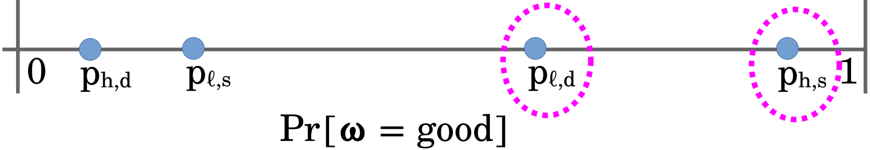
$\left.\begin{array}{l}d = \text{customers like polka dots}\\ s = \text{customers like stripes}\end{array}\right\}$ signal realiz.

$\left.\begin{array}{l}h = \text{“high” marketing campaign}\\ \ell = \text{“low” marketing campaign}\end{array}\right\}$ actions



d = customers like polka dots
s = customers like stripes } signal realiz.
h = “high” marketing campaign
ℓ = “low” marketing campaign } actions

plan: if s then h; if d then ℓ



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s = customers like stripes
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h = “high” marketing campaign
ℓ = “low” marketing campaign
} actions

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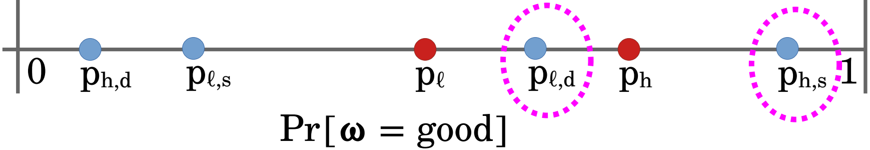

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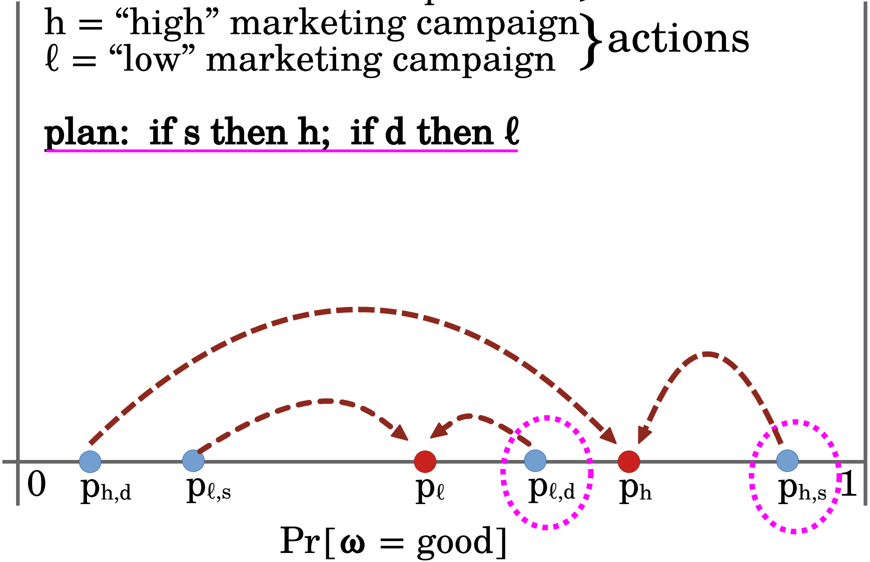

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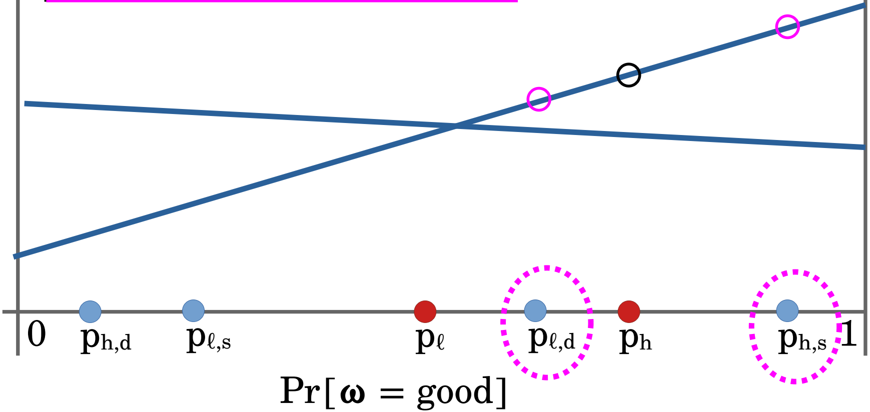

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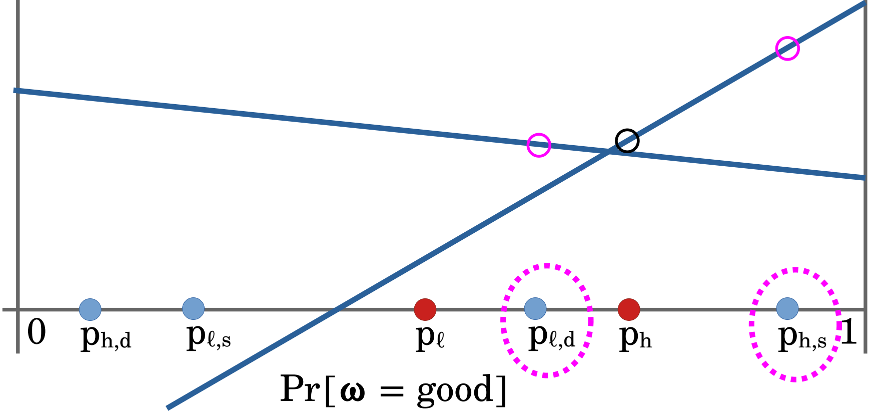


2. Reducing to design of scoring rules

Key characterization

Proposition

WLOG, the menu T is a proper scoring rule $s(p, \omega)$ and the agent reports their posterior belief p in Step 3.

Key characterization

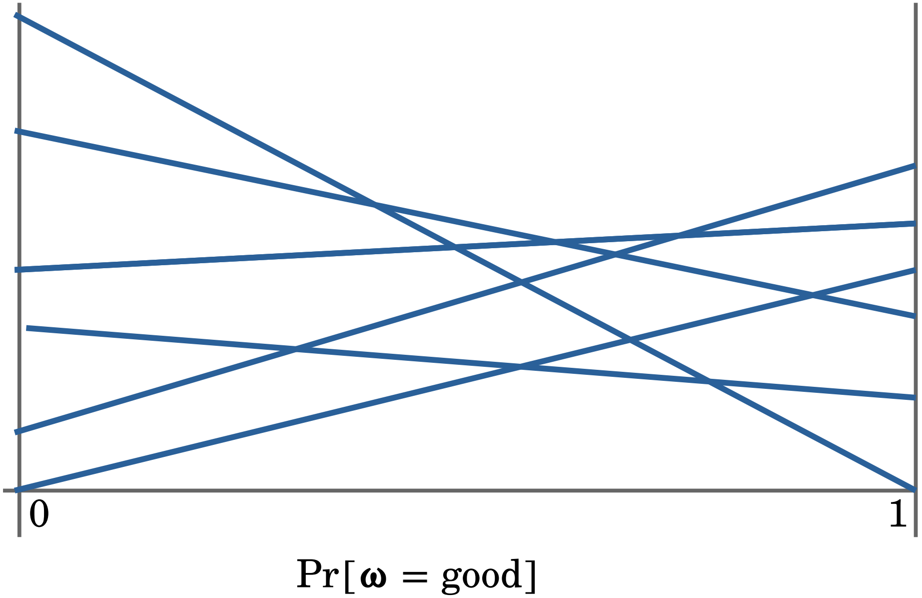
Proposition

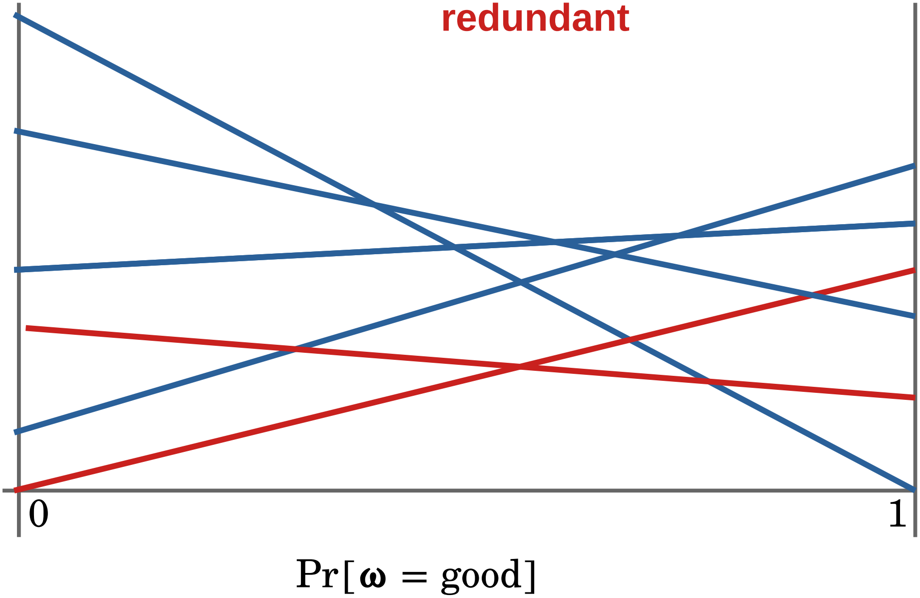
WLOG, the menu T is a proper scoring rule $s(p, \omega)$ and the agent reports their posterior belief p in Step 3.

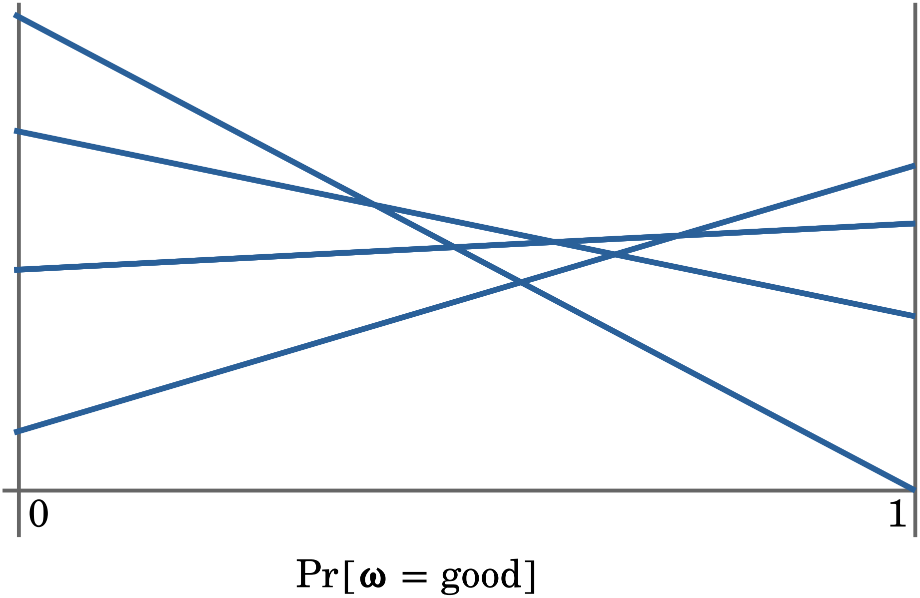
Proposition (Restated)

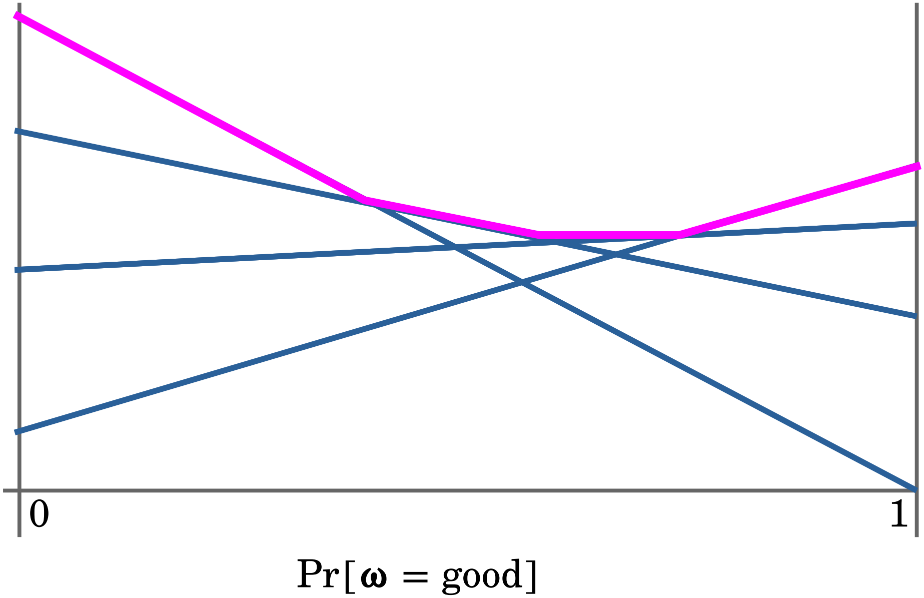
WLOG, the menu T is the set of subtangents of a subdifferentiable convex $G : \Delta_\Omega \to \mathbb{R}$, with

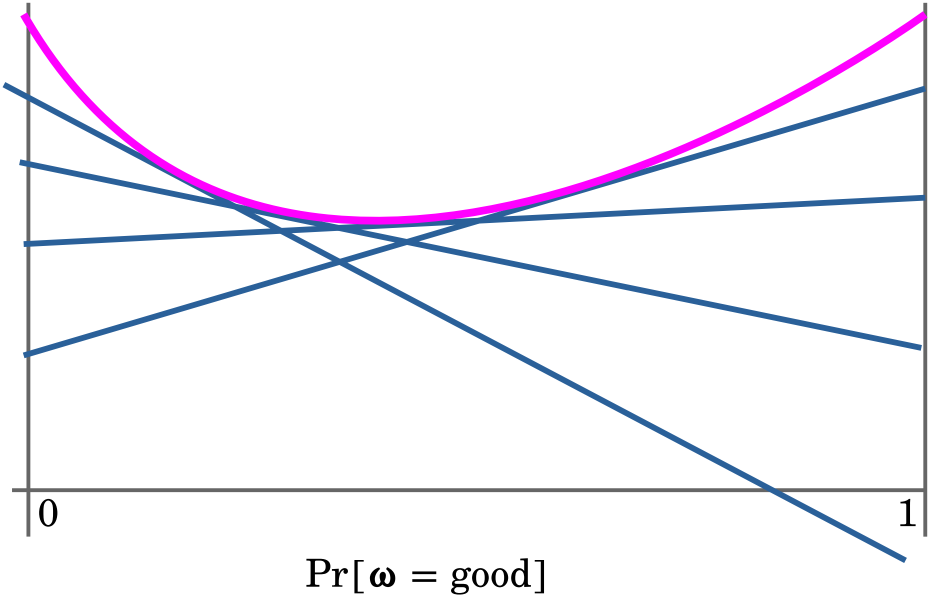
$$G(p) = \max_{t \in T} \bar{t}(p).$$


0
1
Pr[ω = good]


redundant
0
1
Pr[ω = good]


0
1
$\Pr[\omega = \text{good}]$


0
1
Pr[ω = good]


0
1
Pr[ω = good]

3. Special cases

Recovering information acquisition

1. Principal offers menu T
2. Agent chooses whether to acquire signal S
3. Agent selects contract $t \in T$
4. *(Agent does not take action)*
5. Outcome $\omega \sim p_S$ is revealed, pay $t(\omega)$

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Known: under different constraints, "V" shape is optimal [Li, Hartline, Shan, Wu 2020-2022; Chen and Yu 2021].

Information acquisition - results

Theorem

An optimal solution to the IA problem is G^*, where:

1. *Define* $H(p) = \max_\omega \frac{p(\omega)}{p_0(\omega)}$. $p_0 =$ prior
2. *Define* $G^*(p) = \frac{\kappa}{\mathbb{E}\, H(p_S) - 1}\ H(p)$. $\kappa =$ cost of signal

Information acquisition - results

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Observation: G^* is a *pointed polyhedral cone* with its point at p_0.

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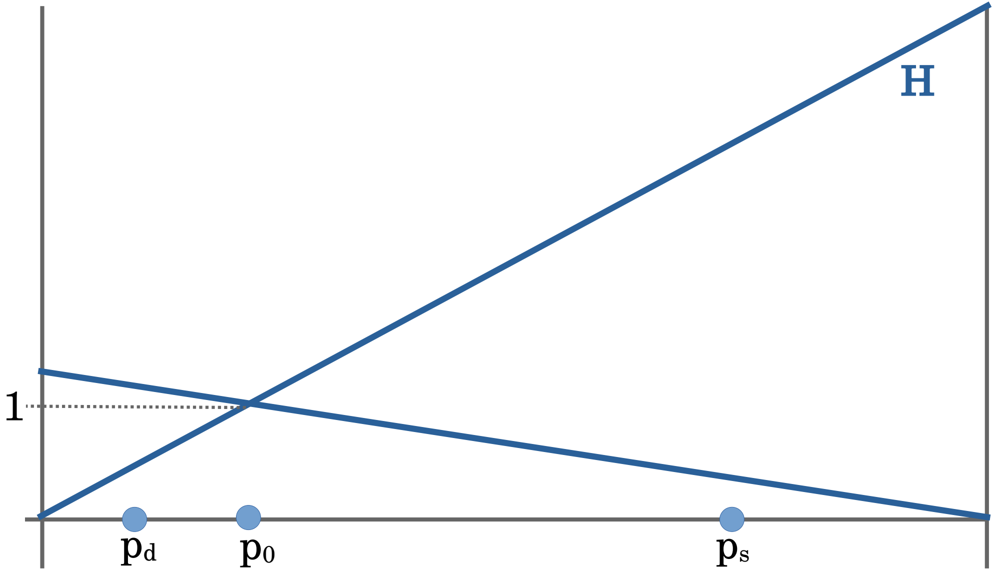
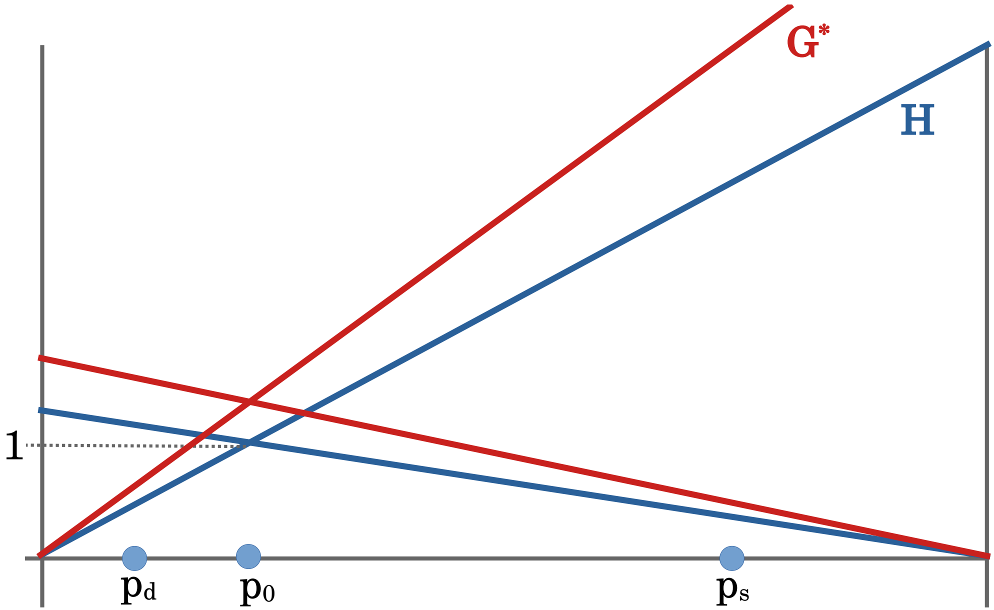
Observation: G^* is a *pointed polyhedral cone* with its point at p_0.

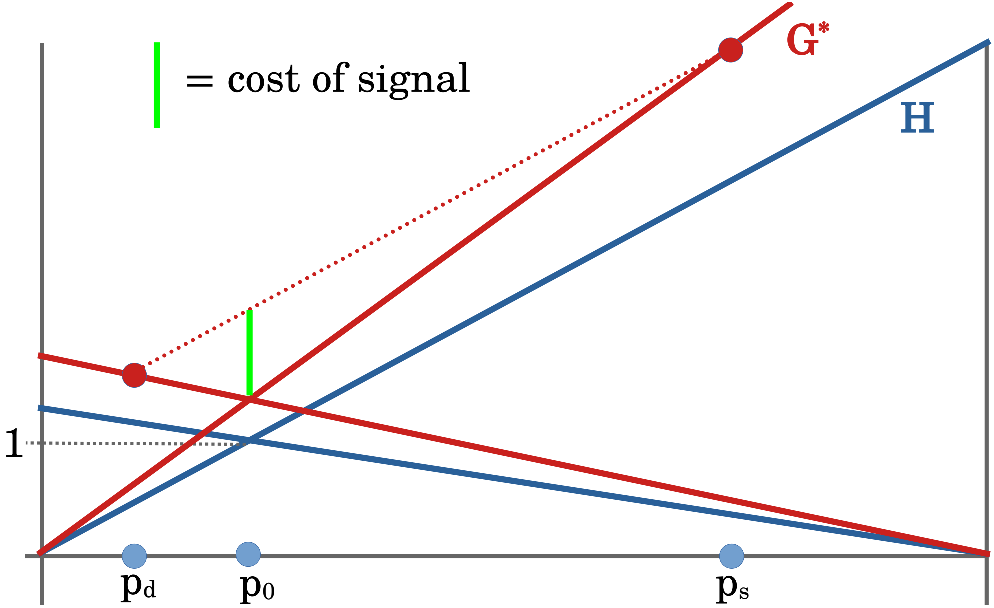
Observation: H contains all "indicator" contracts of the form

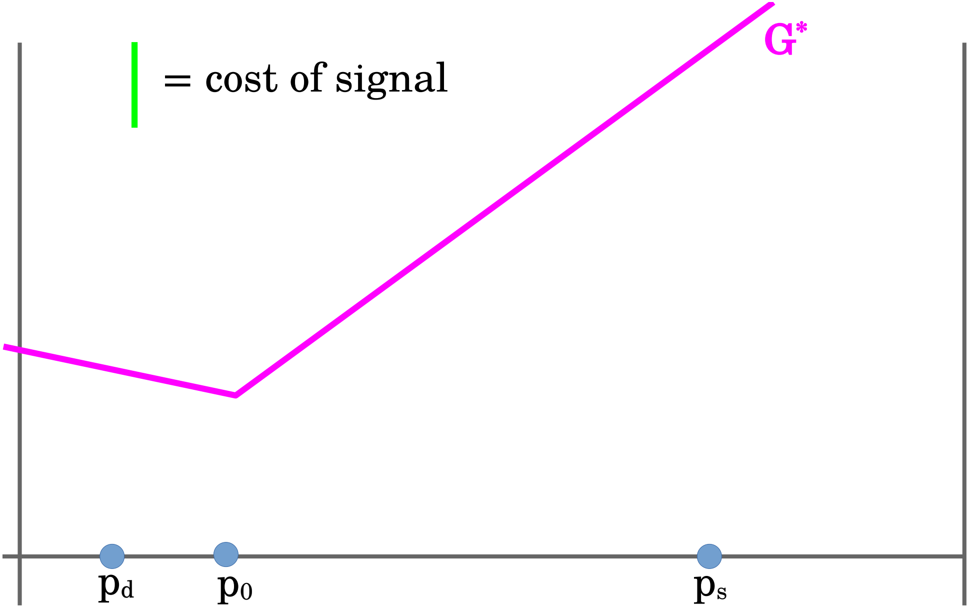
$$t_{\omega^*}(\omega) = \begin{cases} \frac{1}{p_0(\omega^*)} & \omega = \omega^* \\ 0 & \text{otherwise} \end{cases}.$$


H
1
p_d
p_0
p_s



G*
H
1
p_d
p_0
p_s


= cost of signal
G*
H
1
pd
p0
ps


= cost of signal
G*
p_d
p_0
p_s

Proof idea

Lemma: H is feasible (respectively, optimal) on the right $\iff$ $G = \frac{\kappa}{\mathbb{E}\, H - 1} H$ is feasible (respectively, optimal) on the left.

$$\min_{G} \quad \mathbb{E}\, G(p_S)$$

s.t.

$$\mathbb{E}\, G(p_S) - \kappa \geq G(p_0)$$

limited liability

$$\max_{H} \quad \mathbb{E}\, H(p_S)$$

s.t.

$$H(p_0) \leq 1$$

limited liability

Proof idea

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limited liability

$$\max_H \quad \mathbb{E}\,H(p_S)$$

s.t.

$$H(p_0) \leq 1$$

limited liability

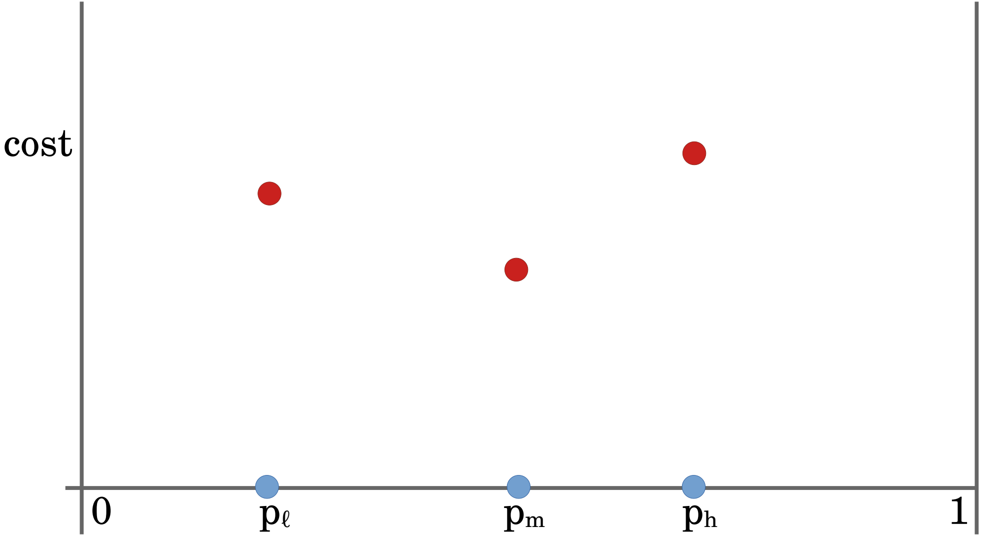
Lemma: $H(p) = \max_\omega \frac{p(\omega)}{p_0(\omega)}$ is optimal on the right.

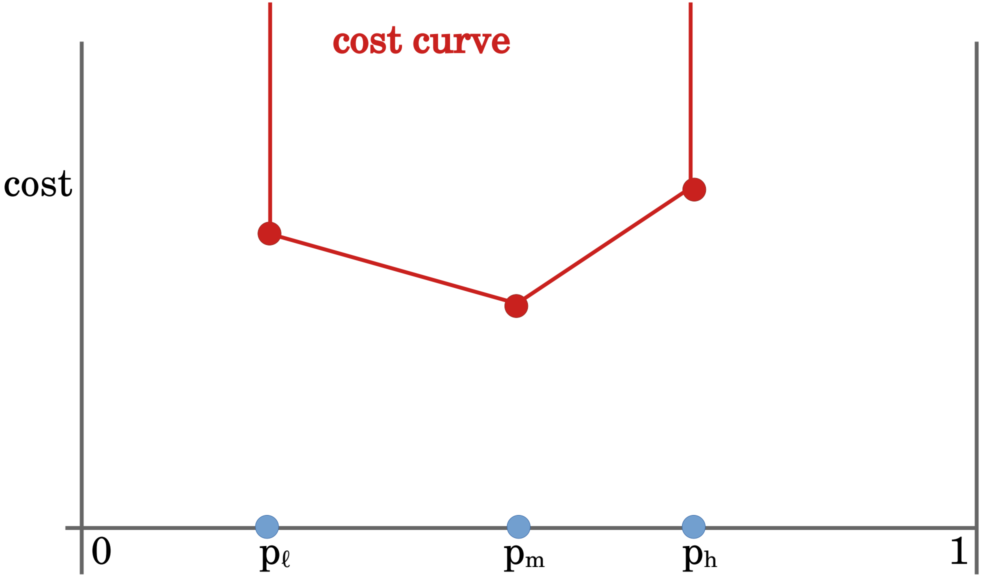
Information acquisition - summary

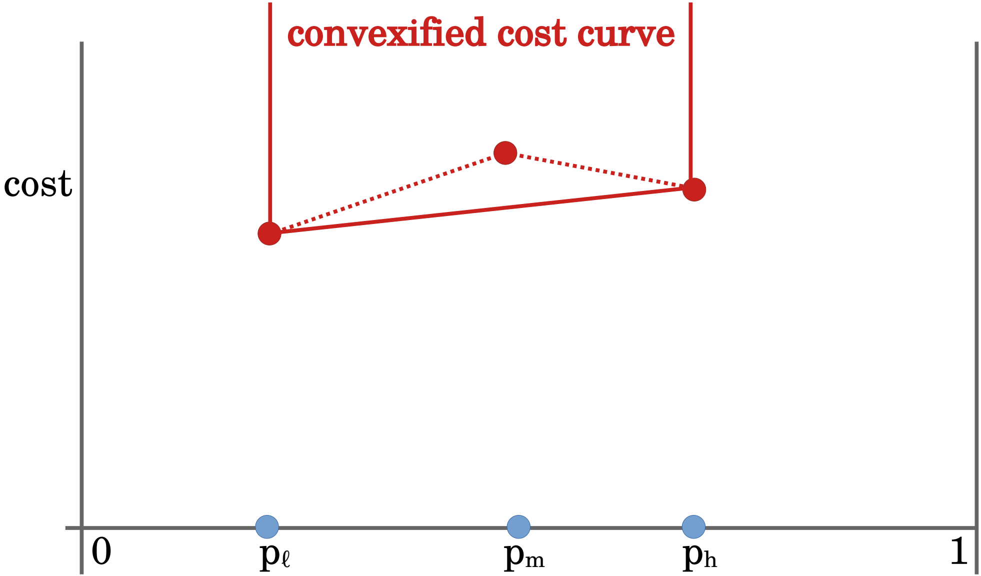
- Solve general multidimensional IA s.t. LL
- $G^* =$ polyhedral pointed cone *as in prior work*
- Closed-form solution

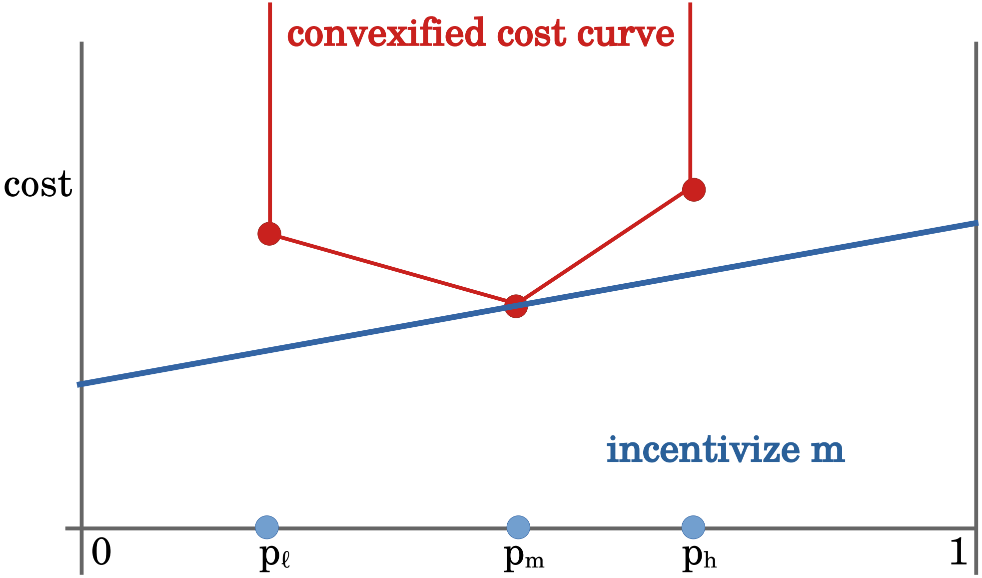
Recovering the hidden action model

1. Principal offers menu T
2. *(there is no signal)*
3. Agent selects contract $t \in T$
4. Agent selects action a
5. Outcome $\omega \sim p_a$ is revealed, pay $t(\omega)$

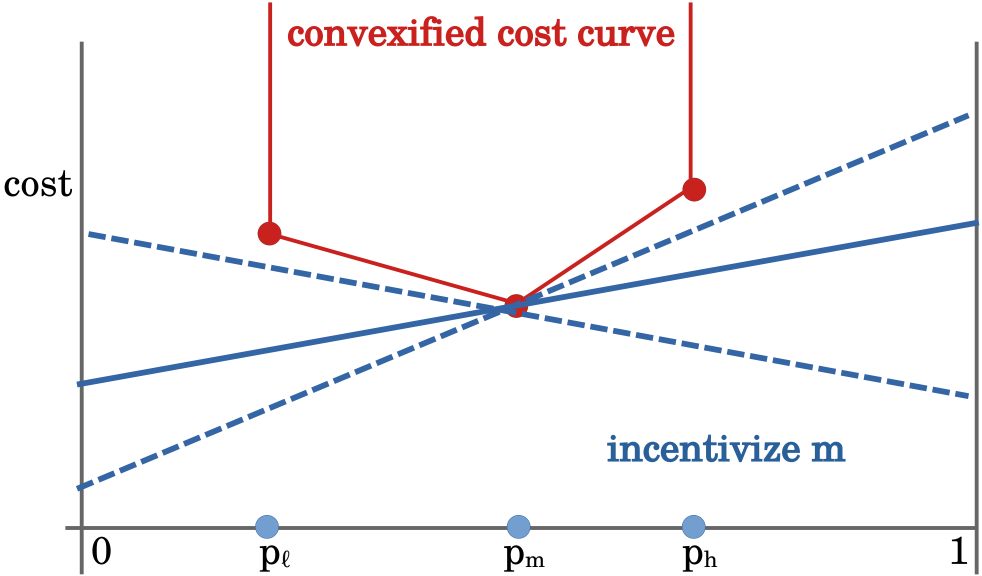

cost
0
p_ℓ
p_m
p_h
1

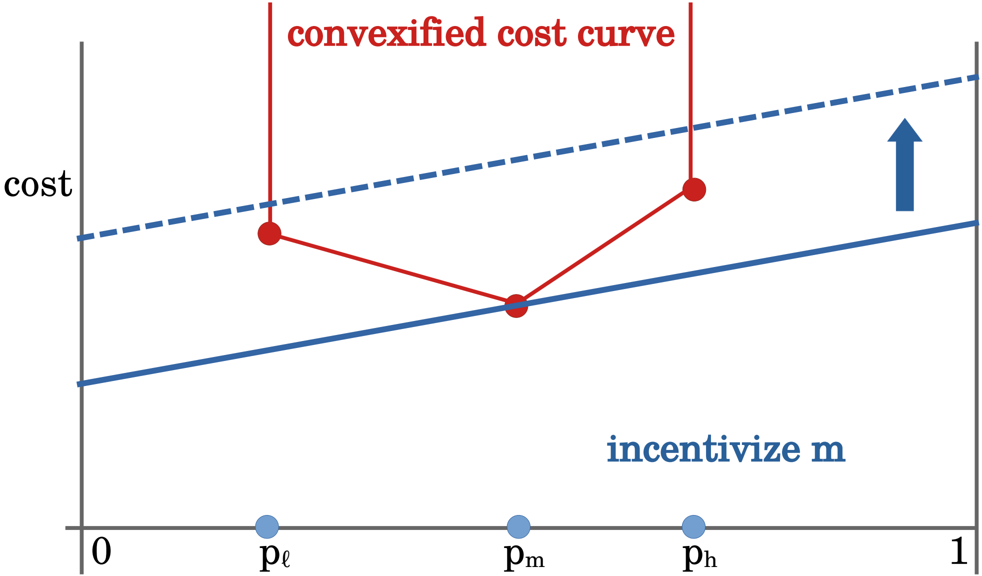

cost curve
cost
0
p_ℓ
p_m
p_h
1

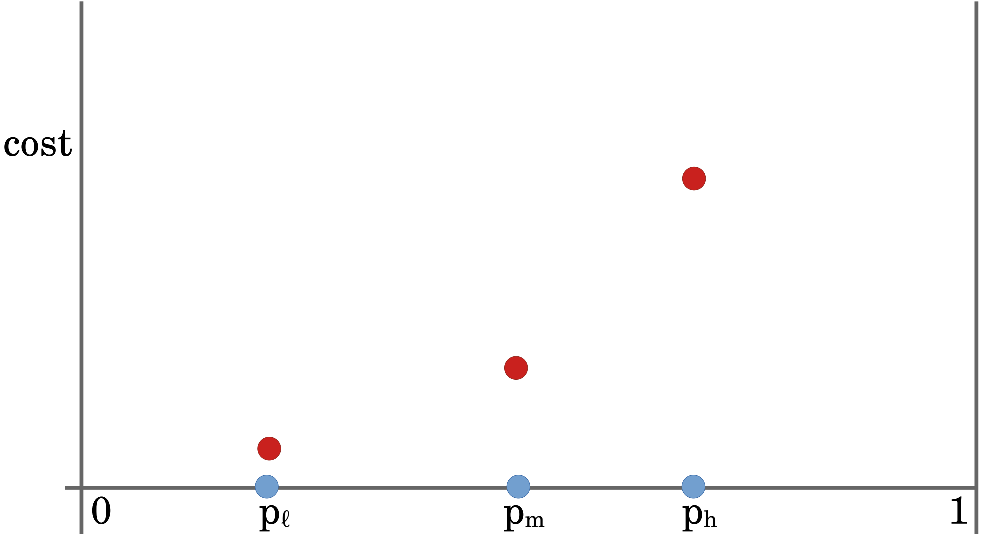

convexified cost curve
cost
0
p_ℓ
p_m
p_h
1

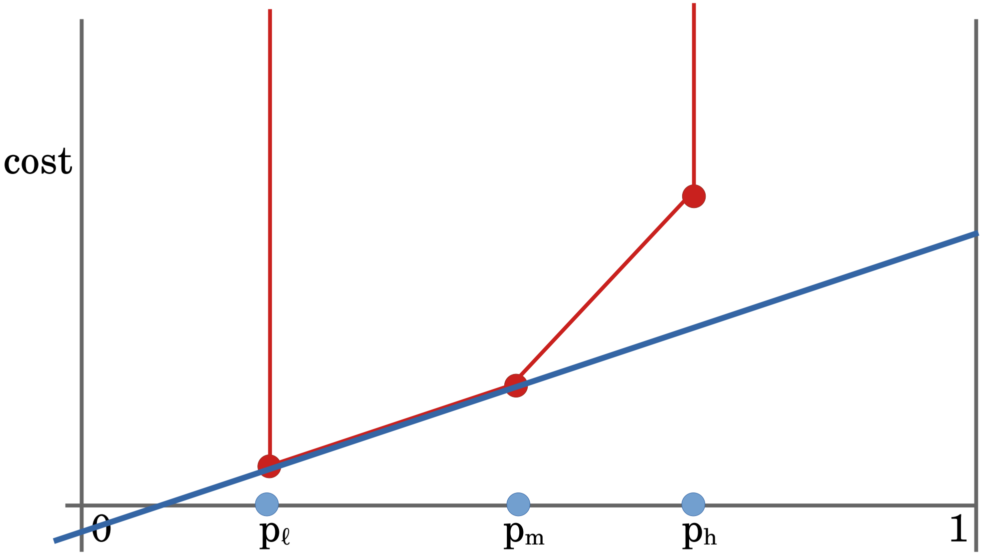


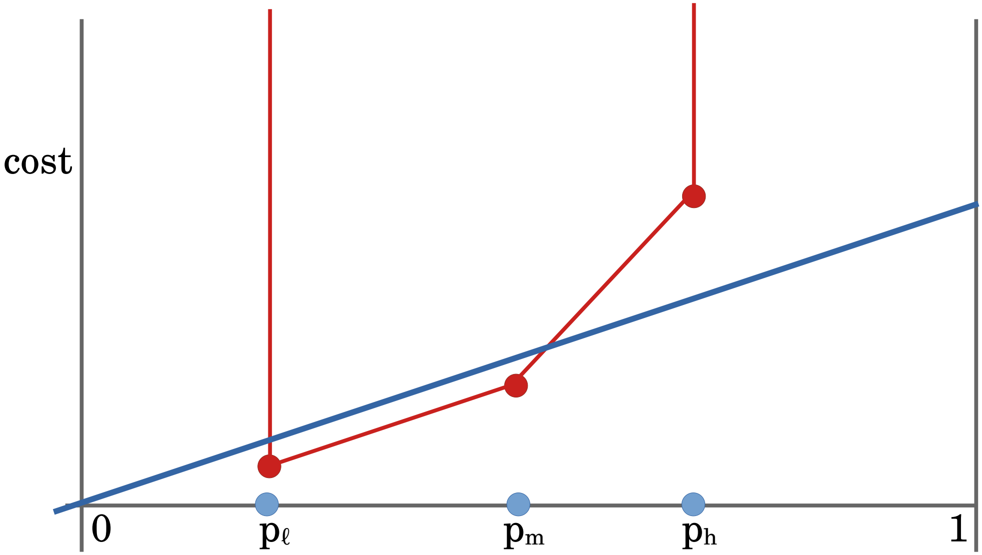
convexified cost curve
cost
incentivize m
0
p_ℓ
p_m
p_h
1


convexified cost curve
cost
incentivize m
0
pℓ
pm
ph
1


convexified cost curve
cost
incentivize m
0
p_ℓ
p_m
p_h
1


cost
0
p_ℓ
p_m
p_h
1


cost
0
p_ℓ
p_m
p_h
1


cost
0
pℓ
pm
ph
1

Hidden actions - summary

- Study the **convexified cost curve**
- Geometric characterization of elicitable actions, optimal contracts
- **But**, no computational advantage over standard LP formulation
- **Still**, useful observations for our general model

4. Main result

Main-ish result

Theorem

For Contracts with Information Acquisition, there is a polynomial-size linear program for computing an optimal menu for a given plan.

Parameters: signal distribution q, action set A, posteriors $\{p_{a,S}\}$, plan $f : S \to A$.

Main-ish result

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Key idea: G is WLOG piecewise linear with a small number of contracts.

Main-ish result

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Key idea: G is WLOG piecewise linear with a small number of contracts.

Extensions: minimizing LP size;
necessary conditions for feasibility of a plan.

Conclusion

Contributions:

- Model and LP for Contracts with Information Acquisition (IA)
- Scoring rule approach to contracts
- Closed-form sol'n for IA under limited liability

Future work:

- Robustness *see Oesterheld+Conitzer 2021*
- Multiple signals *already unknown for IA*
- Efficiently optimize principal utility

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Thanks!